

1. Compute the following derivatives. (3 points each)

a. $C'(q)$ if $C(q) = 32 + .27q + .03q^2$

$$C'(q) = \boxed{0.27 + 0.06q}$$

b. $\frac{d}{dx} [(\ln 2)x]$ (Hint: $\ln 2 \approx .69$)

$$= (\ln 2) \approx \boxed{0.69}$$

c. $\frac{du}{dt}$, if $u = \frac{1}{t} - (t+1)^2$

$$u = \frac{1}{t} - t^2 - 2t - 1$$

$$\frac{du}{dt} = \boxed{-\frac{1}{t^2} - 2t - 2}$$

d. $f''(x)$, if $f(x) = x^7 - x$

$$f'(x) = 7x^6 - 1$$

$$f''(x) = \boxed{42x^5}$$

3. Compute the following limits, or show that the limit does not exist. (3 points each)

a. $\lim_{h \rightarrow 0} \frac{h}{\sqrt{4+h} - \sqrt{4}}$

$$= \lim_{h \rightarrow 0} \frac{h}{\sqrt{4+h} - \sqrt{4}} \cdot \frac{\sqrt{4+h} + \sqrt{4}}{\sqrt{4+h} + \sqrt{4}}$$

$$= \lim_{h \rightarrow 0} \frac{h(\sqrt{4+h} + \sqrt{4})}{h}$$

$$= \lim_{h \rightarrow 0} \sqrt{4+h} + \sqrt{4}$$

$$= \boxed{4}$$

b. $\lim_{x \rightarrow 1} \frac{2x^2 + x - 1}{x + 1}$

$$= \frac{2}{2} \quad (\text{plug in } x=1)$$

$$= \boxed{1}$$

c. $\lim_{x \rightarrow 1} f(x)$, where f is the function

$$f(x) = \begin{cases} x & x < 1 \\ 0 & x = 1 \\ x^2 & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} x^2 = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x = 1$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = \boxed{1}$$

4. (6 points) a. State the limit definition of the derivative. (There are actually two equivalent definitions we learned in class; either is acceptable.)

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

(5 points) b. Use the definition from part a. to compute $\frac{d}{dx} \frac{1}{x+1}$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+1) - (x+h+1)}{(x+h+1)(x+1)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h(x+h+1)(x+1)} \\ &= \lim_{h \rightarrow 0} \frac{-1}{(x+h+1)(x+1)} \\ &= \boxed{\frac{-1}{(x+1)^2}} \end{aligned}$$