1. A bullet is fired directly upwards into the air. The height $H(t)$ of the bullet, in feet, t seconds after it is fired, is given by the formula $H(t) = 2000t - 16t^2$.

a. Compute the average speed of the bullet over the first 10 seconds after it is fired.

$$\text{avg speed} = \frac{H(10) - H(0)}{10} = \frac{20000 - 1600}{10} = 2000 - 160 = 1840 \ (\text{feet/sec})$$

b. Compute the speed of the bullet after 10 seconds.

$$H'(t) = 2000 - 32t$$

$$\text{Speed} = H'(10) = 2000 - 320$$

$$= 1680 \ (\text{feet/sec})$$

2. For each of the following limits, calculate the limit, or show that the limit does not exist:

a. $\lim_{x\to 2} \frac{x^3-1}{x-1}$

$= \lim_{x \to 2} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$

$= \lim_{x \to 2} x^2+x+1$

$= 2^2+2+1$

$= 7$

b. $\lim_{x\to -2} \frac{x^2+6x+8}{x+2}$

$= \lim_{x \to -2} \frac{\cancel{(x+2)}(x+4)}{\cancel{x+2}}$

$= \lim_{x \to -2} x+4$

$= -2+4$

$= 2$

c. $\lim_{x\to 0} \frac{x+x^2}{|x|}$

$\lim_{x \to 0^+} \frac{x+x^2}{|x|} = \lim_{x \to 0^+} \frac{x+x^2}{x} = \lim_{x \to 0^+} 1+x = 1$

$\lim_{x \to 0^-} \frac{x+x^2}{|x|} = \lim_{x \to 0^-} \frac{x+x^2}{-x} = \lim_{x \to 0^-} -1-x = -1$

Since $\lim_{x \to 0^+} \neq \lim_{x \to 0^-}$, $\lim_{x \to 0} \frac{x+x^2}{|x|}$ does not exist

3. Compute the following derivatives.

a. $\frac{d}{dx} e^7$

$= \frac{d}{dx}$ (constant)

$= 0$

b. $\frac{d}{ds} s^5 - 3s^2 + 7$

$= 5s^4 - 6s$

c. $q'(t)$ if $q(t) = \sqrt{t}$

$q(t) = t^{\frac{1}{2}}$

$q'(t) = \frac{1}{2} t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$

d. $\frac{dy}{dx}$ if $y = \frac{x^2+1}{x}$. Hint: first rewrite y as a sum of two different fractions.

$y = x + \frac{1}{x} = x + x^{-1}$

$\frac{dy}{dx} = 1 + -1x^{-2} = 1 - \frac{1}{x^2}$

4. a. Let $f(x) = x^2 - 2x - 3$. Find the x and y intercepts of the graph of f.

x - intercept : $f(x) = 0 \Leftrightarrow x^2 - 2x - 3 = 0$

$\Leftrightarrow (x-3)(x+1) = 0$

$\Leftrightarrow x = 3$ or $x = -1$

$\Rightarrow x = -1$ and $x = 3$ are x-intercepts.

y - intercept : $f(0) = -3 \Rightarrow y = -3$ is y - intercept.

b. Find an equation $y = g(x)$ for the tangent line to the graph of f through the point $(-2, 5)$.

$\frac{dy}{dx} = (x) = 2x - 2$,

$(-2) = -6$ (slope)

By point-slope formula,

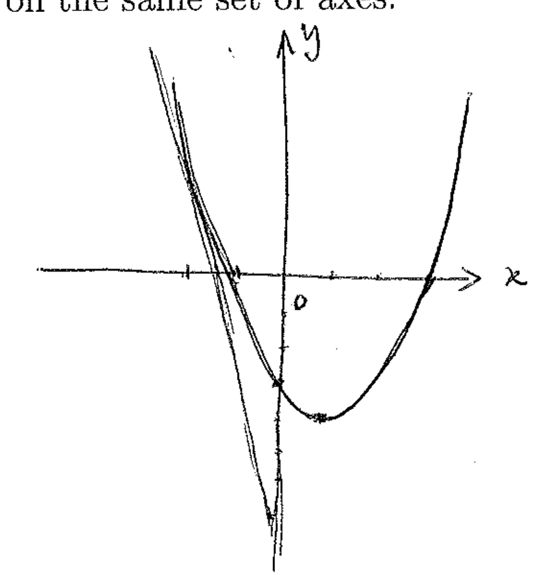
$y - 5 = -6(x - (-2))$

$y - 5 = -6x - 12$

$y = -6x - 7$ is the eqn for the tangent line.

$g(x) = -6x - 7$.

c. Graph f and g on the same set of axes.



5. a. State the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

b. Use the definition from part a. to compute $\frac{d}{dx}\frac{1}{\sqrt{x}}$.

$$\left(\frac{1}{\sqrt{x}}\right)' = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h\sqrt{x+h}\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\sqrt{x+h}\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x+h}\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= -\frac{1}{2x\sqrt{x}}, \quad \text{provided } x > 0.$$

6. The cost $C(x)$, in dollars, to refine x tons of sugar is given by the formula

$$C(x) = 1000 + 20x + .01x^2.$$

How much sugar should be refined in order to minimize the average cost?

$$\text{average cost} = \frac{C(x)}{x} = \frac{1000 + 20x + 0.01x^2}{x}$$

$$= \frac{1000}{x} + 20 + 0.01x$$

$$\frac{d}{dx}\left(\frac{C(x)}{x}\right) = -\frac{1000}{x^2} + 0.01$$

$$= \frac{-1000 + 0.01x^2}{x^2}$$

$$= \frac{0.01(x^2 - 100000)}{x^2}$$

$$\begin{cases} <0, & x < \sqrt{100000} \\ >0, & x > \sqrt{100000} \end{cases}$$

$\Rightarrow$ when $x = \sqrt{100000} = 100\sqrt{10}$ (tons)

the average cost is minimized.