1. Find the tangent line to the ellipse $x^2 - xy + y^2 = 1$ through the point $(0, 1)$.

known $(0,1)$ on the line,

only need to find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left(x^2 - xy + y^2\right) = \frac{d}{dx}(1)$$

$$2x - \left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$2x - y = (x - 2y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=1}} = \frac{0-1}{0-2} = \frac{1}{2}$$

eqn: $y - 1 = \frac{1}{2}(x - 0)$

$$\boxed{y = \frac{1}{2}x + 1}$$

2. a. Find the absolute maximum and minimum value of $2\ln x - x$ on $[1,3]$, together with the location(s) where each is attained (if they exist).

$$\frac{d}{dx}\left(2\ln x - x\right) = \frac{2}{x} - 1 = 0$$

$$\Rightarrow \frac{2}{x} = 1 \Rightarrow x = 2 \quad \left(\text{inside } [1,3]\right)$$

$f(1) = -1$

$f(2) = 2\ln 2 - 2 \approx -0.6$

$f(3) = 2\ln 3 - 3 \approx -0.8$

max @ $(2, 2\ln 2 - 2)$

min @ $(1, -1)$

b. Find the absolute maximum and minimum value of $\frac{x^2-x+1}{x+1}$ on $[0,2]$, together with the location(s) where each is attained (if they exist).

$$\frac{d}{dx}\left(\frac{x^2-x+1}{x+1}\right) = \frac{(2x-1)(x+1)-(x^2-x+1)}{(x+1)^2} = \frac{(2x^2+x-1)-(x^2-x+1)}{(x+1)^2}$$

$$= \frac{x^2+2x-2}{(x+1)^2} = \begin{cases} 0 \Rightarrow x = -1+\sqrt{3},\ -1-\sqrt{3}\ (\text{not in } [0,2]) \\ \text{undefined} \Rightarrow x = -1,\ \text{not in } [0,2] \end{cases}$$

critical point: $x = -1+\sqrt{3}$

$f(0) = 1$

$f(2) = 1$

$f(-1+\sqrt{3}) = 2\sqrt{3} - 3$

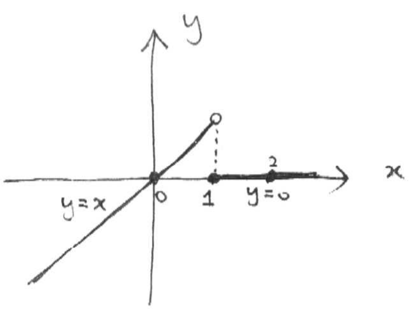
max @ $(0,1)$, $(2,1)$

min @ $(-1+\sqrt{3}, 2\sqrt{3}-3)$

c. Find the absolute maximum and minimum value of

$$g(x) = \begin{cases} x & x < 1 \\ 0 & x \geq 1 \end{cases}$$

on $[0,2]$, together with the location(s) where each is attained (if they exist). Hint: draw the graph first.



min @ $(0,0)$, $(x,0)$, $1 \leq x \leq 2$.

max DNE.

3. Graph the function $f(x) = xe^x$, being sure to clearly indicate all critical points, local maximum and minimum values, inflection points, and vertical and horizontal asymptotes.

$$f'(x) = e^x + xe^x = (1+x)e^x = 0$$

$$\Rightarrow \quad x = -1$$

inc/dec :

f	↘	↗
f'	$- - - - -$	$+ + + + +$
	-1	

critical pt: $x = -1$

loc min @ $(-1, -e^{-1})$

loc max DNE.

$$f''(x) = e^x + (e^x + xe^x) = 2e^x + xe^x = (2+x)e^x$$

$$= 0 \quad \Rightarrow \quad x = -2$$

concavity :

f	$\frown$	$\smile$
f''	$- - - - -$	$+ + + + +$
	-2	

inflection pt @ $(-2, -2e^{-2})$

$$\lim_{x \to \infty} xe^x = \infty \cdot \infty = \infty$$

$$\lim_{x \to -\infty} xe^x = 0 \quad \text{(use calculator)}$$

So $y = 0$ is a horizontal asympt.

No vertical asympt. because the function is continuous.