

1. (10 pts) The demand function for an electronics company's car stereos is $D(q) = 2 - q$ and the supply function is $S(q) = q^2$, where q is measured in thousands.
- At what price is the market for the stereos in equilibrium?
 - What is the maximum total surplus?

$$\begin{aligned} \text{a. } D(q) = S(q) &\Rightarrow 2 - q = q^2 \Rightarrow q^2 + q - 2 = 0 \Rightarrow (q+2)(q-1) = 0 \\ &\Rightarrow \boxed{q = 1} \text{ or } q = -2 \quad (q \geq 0) \end{aligned}$$

$D(1) = 1 \Rightarrow$ at price $\boxed{p = 1}$ the market is in equilibrium.

$$\begin{aligned} \text{b. } \int_0^1 (2 - q) - (q^2) dq &= \int_0^1 2 - q - q^2 dq = \left[2q - \frac{q^2}{2} - \frac{q^3}{3} \right]_0^1 \\ &= 2 - \frac{1}{2} - \frac{1}{3} = 2 - \frac{5}{6} = \boxed{\frac{7}{6}} \end{aligned}$$

2. (10 pts) A retiree is paid \$1500 per month by an annuity. If the income is invested in an account that earns 5% interest compounded continuously.
- What is the future value of the income after ten years?
 - What is the present value of the income over a ten-year period?

$$\begin{aligned} \text{a. } FV &= e^{+0.05 \cdot 10} \int_0^{10} 1500 e^{-0.05t} dt = e^{+0.5} 1500 \int_0^{10} e^{-0.05t} dt \\ &= e^{+0.5} 1500 \left[\frac{e^{-0.05t}}{-0.05} \right]_0^{10} = e^{+0.5} 1500 \left[\frac{e^{-0.5}}{-0.05} - \frac{1}{-0.05} \right] \\ &= e^{+0.5} \frac{1500}{0.05} (1 - e^{-0.5}) = \boxed{e^{+0.5} 30000 (1 - e^{-0.5})} = 30000 (e^{0.5} - 1) \end{aligned}$$

$$\text{b. } PV = \boxed{30000 (1 - e^{-0.5})}$$