Name: ______________________ Math 211 Quiz 11

Section: 322 ☐ 323 ☐ Dec 13, 2012

1. (10 pts) Find the solution of the differential equation that satisfies the given initial condition.

a. $\frac{dy}{dx} = -\frac{x}{y}$, $y(0) = 1$ b. $y' = -2xy$, $y(0) = 1$

a.

$y\,dy = -x\,dx$

$\int y\,dy = -\int x\,dx$

$\frac{y^2}{2} = -\frac{x^2}{2} + C$

$y^2 = -x^2 + C$

$y = \pm\sqrt{-x^2 + C}$

$y(0) = 1$

$\Rightarrow \pm\sqrt{0 + C} = 1$

$\Rightarrow$ "$\pm$" = "+", $C = 1$

$\Rightarrow y = \boxed{\sqrt{-x^2+1}}$

b.

$\frac{dy}{dx} = -2xy$

$\frac{dy}{y} = -2x\,dx$

$\int \frac{dy}{y} = -2\int x\,dx$

$\ln|y| = -2\frac{x^2}{2} + C$

$\ln|y| = -x^2 + C$

$e^{\ln|y|} = e^{-x^2+C}$

$|y| = Ce^{-x^2}$ $(C > 0)$

$y = Ce^{-x^2}$, $C \neq 0$

$y(0) = 1$

$\Rightarrow Ce^{-0} = 1$

$\Rightarrow C = 1$

$\Rightarrow y = \boxed{e^{-x^2}}$

2. (10 pts) Evaluate the improper integral.

a. $\int_0^\infty e^{-0.05t}\,dt$ b. $\int_{-\infty}^{-1} \frac{1}{x^2}\,dx$

a.

$\lim_{T\to\infty} \int_0^T e^{-0.05t}\,dt$

$= \lim_{T\to\infty} \left[\frac{e^{-0.05t}}{-0.05}\right]_0^T$

$= \lim_{T\to\infty} \left[\frac{e^{-0.05T}}{0.05} - \frac{1}{-0.05}\right]$

$= 0 + \frac{1}{0.05}$

$= \boxed{20}$

b.

$\lim_{T\to\infty} \int_{-T}^{-1} x^{-2}\,dx$

$= \lim_{T\to\infty} \left[\frac{x^{-1}}{-1}\right]_{-T}^{-1}$

$= \lim_{T\to\infty} \left[1 - \frac{(-T)^{-1}}{-1}\right]$

$= \lim_{T\to\infty} \left[1 - \frac{1}{T}\right]$

$= 1 - 0$

$= \boxed{1}$