

1. (10 pts) Find the derivatives of the following functions and simplify your answers.

a.  $g(t) = (\sqrt{t} + 1)\left(\frac{1}{\sqrt{t}} + 1\right)$       b.  $f(t) = \frac{3e^t}{t^2 + 2t + 2}$

a.  $\frac{1}{2\sqrt{t}} \left( \frac{1}{\sqrt{t}} + 1 \right) + (\sqrt{t} + 1) \left( -\frac{1}{2\sqrt{t}^3} \right)$       b.  $\frac{3e^t(t^2 + 2t + 2) - 3e^t(2t + 2)}{(t^2 + 2t + 2)^2}$

$= \frac{1}{2t} + \frac{1}{2\sqrt{t}} + \cancel{-\frac{1}{2t}} - \frac{1}{2\sqrt{t}^3}$

$= \boxed{\frac{1}{2\sqrt{t}} - \frac{1}{2\sqrt{t}^3}}$

$= \frac{3e^t(t^2 + \cancel{2t} + 2 - \cancel{2t} - 2)}{(t^2 + 2t + 2)^2}$

$= \boxed{\frac{3e^t t^2}{(t^2 + 2t + 2)^2}}$

2. (10 pts) A car-detailing service estimates that its daily cost of waxing  $q$  cars is

$$C(q) = 0.08q^2 + 37q + 350.$$

If the service collects \$65 for each car waxing, find

a. (3 pts) the revenue function  $R(q)$ .

b. (7 pts) the number of cars the service should wax daily in order to maximize profit.

a.  $R(q) = q \cdot \text{price} = \boxed{65q}$

b.  $C'(q) = R'(q)$

$\Rightarrow 0.16q + 37 = 65$

$\Rightarrow 0.16q = 28$

$q = \frac{28}{0.16} = \frac{2800}{16} = \frac{7 \cdot 4 \cdot 100}{4 \cdot 4} = 7 \cdot 25 = \boxed{175}$

Conclusion: should wax 175 cars (if possible) to maximize profit.