

1. (10 pts) Find the derivatives of the following functions.

a. $f(x) = \sqrt{x^2 + 2x + 1}$

b. $g(t) = e^{-1/t^2}$

a. $f'(x) = \frac{1}{2} \frac{2x+2}{\sqrt{x^2+2x+1}}$

b. $g'(t) = e^{-1/t^2} (2t^{-3})$

$$= \frac{x+1}{\sqrt{x^2+2x+1}}$$

$$= \frac{2e^{-1/t^2}}{t^3}$$

$$= \frac{x+1}{\sqrt{(x+1)^2}}$$

$$= \frac{x+1}{|x+1|}$$

$$= \operatorname{sgn}(x+1)$$

2. (5 pts) Find dy/dx by implicit differentiation.

$$\ln(xy^2) = x^2 - y$$

$$\frac{1}{xy^2} (y^2 + x2yy') = 2x - y'$$

$$\frac{1}{x} + \frac{2}{y} y' = 2x - y'$$

$$\left(\frac{2}{y} + 1\right) y' = 2x - \frac{1}{x}$$

$$y' = \frac{2x - \frac{1}{x}}{\frac{2}{y} + 1}$$

3. (5 pts) How long does it take for a population that is growing with a constant relative growth rate at 10% to triple?

$$\text{population after time } t = P e^{0.1t}$$

$$P e^{0.1t} = 3P$$

$$e^{0.1t} = 3$$

$$0.1t = \ln 3$$

$$t = \boxed{10 \ln 3}$$