Name: ______________________________ Math 211 Quiz 9

Section: 322 ☐ 323 ☐ Nov 29, 2012

1. (10 pts) Use the substitution rule to evaluate the integral.

a. $\int \frac{t}{\sqrt{1+t^2}}dt$ b. $\int_0^1 xe^{1-x^2}dx$

a. $u = 1+t^2$

$du = 2t\,dt$

$dt = \frac{du}{2t}$

$\int \frac{\cancel{t}}{\sqrt{u}} \frac{du}{2\cancel{t}}$

$= \int \frac{1}{2\sqrt{u}} du$

$= \sqrt{u} + C$

$= \boxed{\sqrt{1+t^2} + C}$

b. $u = 1-x^2$

$du = -2x\,dx$

$dx = \frac{du}{-2x}$, $u(0) = 1$, $u(1) = 0$

$\int_{u(0)}^{u(1)} x\, e^u \frac{du}{-2x}$

$= \int_1^0 \frac{e^u}{-2} du$

$= \left.\frac{e^u}{-2}\right]_1^0$

$= \frac{1}{-2} - \frac{e}{-2} = \boxed{\frac{e-1}{2}}$

2. (10 pts) Use integration by parts to evaluate the integral.

a. $\int \frac{t}{e^t}dt$ b. $\int_1^e x \ln x\,dx$

a. $= \int t e^{-t} dt$

$= \int t\, d(-e^{-t})$

$= t(-e^{-t}) - \int -e^{-t} dt$

$= -te^{-t} + \frac{e^{-t}}{-1} + C$

$= \boxed{-te^{-t} - e^{-t} + C}$

b. $= \int_1^e \ln x\, d\left(\frac{x^2}{2}\right)$

$= \left.(\ln x)\frac{x^2}{2}\right]_1^e - \int_1^e \frac{x^2}{2}\frac{1}{x}dx$

$= \left(\frac{e^2}{2} - 0\right) - \int_1^e \frac{x}{2}dx$

$= \frac{e^2}{2} - \left.\frac{x^2}{4}\right|_1^e$

$= \frac{e^2}{2} - \left(\frac{e^2}{4} - \frac{1}{4}\right)$

$= \boxed{\frac{e^2}{4} + \frac{1}{4}}$