Name: ______________________________ Math 211 Quiz 11

Section: 302 ☐ 303 ☐ Apr 25, 2012

1. Find the indicated integral.

$$\text{a. } \int (x+1)^9 dx \quad \text{b. } \int \frac{1}{x-1} dx \quad \text{c. } \int \sqrt{2x+1} dx \quad \text{d. } \int \frac{x}{2x+1} dx$$

a. $u = x+1$, $du = dx$, $\int u^9 du = \frac{u^{10}}{10} + C = \boxed{\frac{(x+1)^{10}}{10} + C}$

b. $u = x-1$, $du = dx$, $\int \frac{1}{u} du = \ln|u| + C = \boxed{\ln|x-1| + C}$

c. $u = 2x+1$, $du = 2dx$, $dx = \frac{1}{2} du$, $\int \sqrt{u} \frac{1}{2} du = \frac{1}{2} \int u^{\frac{1}{2}} du = \frac{1}{2} \frac{u^{3/2}}{3/2} + C = \boxed{\frac{1}{3}(2x+1)^{3/2} + C}$

d. $u = 2x+1$, $dx = \frac{1}{2} du$, $\int \frac{x}{u} \frac{1}{2} du = \int \frac{u-1}{2} \frac{1}{u} \frac{1}{2} du = \frac{1}{4} \int \frac{u-1}{u} du$

$x = \frac{u-1}{2}$

$= \frac{1}{4} \int 1 - \frac{1}{u} \, du = \frac{1}{4}(u - \ln|u|) + C$

$= \boxed{\frac{1}{4}(2x+1 - \ln|2x+1|) + C}$

2. Find the indicated integral.

$$\text{a. } \int t\sqrt{t^2+1} dt \quad \text{b. } \int t^2(t^3-1)^5 dt \quad \text{c. } \int x^4 e^{x^5-1} dx \quad \text{d. } \int \frac{\ln x}{x} dx$$

a. $u = t^2+1$, $du = 2t\,dt$, $dt = \frac{1}{2t} du$, $\int t\sqrt{u} \frac{1}{2t} du = \frac{1}{2} \int \sqrt{u}\, du = \frac{1}{3} u^{3/2} + C$

$= \boxed{\frac{1}{3}(t^2+1)^{3/2} + C}$

c. $u = x^5 - 1$, $du = 5x^4 dx$, $dx = \frac{1}{5x^4} du$,

$\int x^4 e^u \frac{1}{5x^4} du = \frac{1}{5} \int e^u du = \frac{1}{5} e^u + C = \boxed{\frac{1}{5} e^{x^5-1} + C}$

b. $u = t^3 - 1$, $du = 3t^2 dt$, $dt = \frac{1}{3t^2} du$

$\int t^2 u^5 dt = \int t^2 u^5 \frac{1}{3t^2} du = \frac{1}{3} \int u^5 du = \frac{1}{3} \frac{u^6}{6} + C = \boxed{\frac{1}{18}(t^3-1) + C}$

d. $u = \ln x$, $du = \frac{1}{x} dx$, $dx = x\,du$

$\int \frac{u}{x} x\,du = \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(\ln x)^2}{2} + C}$

Bonus problem. Solve the given initial value problem for $y = f(x)$.

$$\frac{dy}{dx} = \frac{x+2}{x^2+4x+5}, y(-1) = 3$$

$$y = \int \frac{x+2}{x^2+4x+5}\,dx \quad \frac{u = x^2+4x+5}{\substack{du = (2x+4x)dx \\ = 2(x+2)dx \\ dx = \frac{1}{2(x+2)}du}} \quad \int \frac{\cancel{x+2}}{u} \frac{1}{2\cancel{(x+2)}}\,du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+4x+5| + C$$

$$3 = y(-1) = \frac{1}{2}\ln|1-4+5| + C = \frac{1}{2}\ln 2 + C$$

$$\Rightarrow C = 3 - \frac{1}{2}\ln 2.$$

$$y(x) = \frac{1}{2}\ln|x^2+4x+5| + 3 - \frac{1}{2}\ln 2$$

$$= \frac{1}{2}\left[\ln|x^2+4x+5| - \ln 2\right] + 3$$

$$= \boxed{\frac{1}{2}\ln\left|\frac{x^2+4x+5}{2}\right| + 3}.$$