Name: ______________________ Math 211 Quiz 12

Section: 302 ☐ 303 ☐ May 2, 2012

1. Evaluate the given definite integral.

$$\text{a. } \int_4^9 (\sqrt{x}-1)dx \quad \text{b. } \int_0^1 (e^x - e^{-x})dx \quad \text{c. } \int_1^{e^2} \frac{(\ln x)^2}{x} dx$$

a. $\left.\frac{x^{3/2}}{3/2}\right|_4^9 - \left. x \right|_4^9 = \frac{2}{3}(27-8) - (9-4) = \frac{38}{3} - 5 = \boxed{\frac{23}{3}}$

b. $\left.(e^x + e^{-x})\right|_0^1 = (e + e^{-1}) - (2) = \boxed{e + e^{-1} - 2}$

c. $(u = \ln x) = \int_0^2 u^2\, du = \left.\frac{u^3}{3}\right|_0^2 = \boxed{\frac{8}{3}}$

2. Find the average value of the given function over the the specified interval.

$$\text{a. } 1 - x^2, -3 \le x \le 3 \quad \text{b. } \frac{x+1}{x^2+2x+2}, -1 \le x \le 1$$

a. $\frac{1}{6}\int_{-3}^{3} (1-x^2)\, dx = \frac{1}{6}\left.\left(x - \frac{x^3}{3}\right)\right|_{-3}^{3} = \frac{1}{6}\left((3-9) - (-3+9)\right)$

$= \frac{1}{6}(-12) = \boxed{-2}$

b. $\frac{1}{2}\int_{-1}^{1} \frac{x+1}{x^2+2x+2}\, dx \overset{u = x^2+2x+2}{=\!=} \frac{1}{2}\int_{u(-1)}^{u(1)} \frac{1}{2u}\, du = \frac{1}{4}\left.\left(\ln|u|\right)\right|_1^5$

$= \boxed{\frac{1}{4}\ln 5}$

3. Sketch the region between the curve $y = x^2 - 2x$ and $y = -x^2 + 4$ and then find its area.

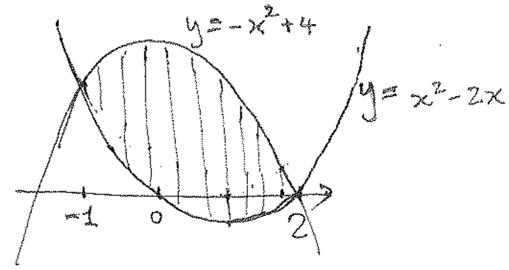
$$x^2 - 2x = -x^2 + 4$$

$$\Rightarrow \quad 2x^2 - 2x - 4 = 0$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad (x-2)(x+1) = 0$$

$\Rightarrow \quad x = -1$ or 2 (intersection pts)



$$\text{area} = \int_{-1}^{2} \left[(-x^2+4) - (x^2-2x)\right] dx$$

$$= \int_{-1}^{2} -2x^2 + 2x + 4 \, dx$$

$$= -2\frac{x^3}{3}\Big|_{-1}^{2} + 2\frac{x^2}{2}\Big|_{-1}^{2} + 4x\Big|_{-1}^{2}$$

$$= -\frac{2}{3}(8+1) + (4-1) + 12$$

$$= -6 + 3 + 12$$

$$= \boxed{9}$$