Name: ______________________________ Math 211 Quiz 3

Section: 302 ☐ 303 ☐ Feb 8, 2012

1. Find the indicated limit. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$.

a. $\lim_{x\to 1} \frac{x^2+4x-5}{x^2-1}$ **b.** $\lim_{x\to\infty} \frac{2x^2-3x}{x^2-1}$ **c.** $\lim_{x\to-\infty} \frac{x^3+x}{2x^2+1}$ **d.** $\lim_{x\to 1^-} \frac{1}{x-1}$

a. $= \lim_{x\to 1} \frac{(x+5)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \frac{6}{2} = 3$

b. $= \lim_{x\to\infty} \frac{2x^2}{x^2} = 2$

c. $= \lim_{x\to-\infty} \frac{x^3}{2x^2} = \lim_{x\to-\infty} \frac{x}{2} = -\infty$

d. $= \frac{1}{0^-} = -\infty$

2. Compute the derivative of the given function and find the slope of the line that is tangent to its graph for the specified value of the independent variable.

a. $f(x) = x^2 - 1;\ \ x = -1$ **b.** $g(t) = \sqrt{t};\ \ t = 4$

a. $f'(x) = \frac{d}{dx}x^2 - \frac{d}{dx}1$

$= 2x - 0$

$= 2x$

Slope at -1 is $f'(-1) = -2$

b. $g'(t) = \frac{d}{dt}\sqrt{t}$

$= \frac{1}{2}\frac{1}{\sqrt{t}}$

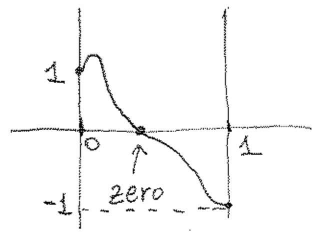
Slope at $4 = g'(4) = \frac{1}{4}$

* For the proofs of $\frac{d}{dx}x^2 = 2x$ and $\frac{d}{dt}\sqrt{t} = \frac{1}{2\sqrt{t}}$, see the textbook.

3. Show that the equation $\sqrt{x} = x^2 + 2x - 1$ must have at least one solution on the interval $0 \le x \le 1$. *hint: use intermediate value property of continuous functions.*

$$\sqrt{x} = x^2 + 2x - 1$$

$$\Leftrightarrow \; f(x) = \sqrt{x} - x^2 - 2x + 1 = 0$$



$$f(1) = 1 - 1 - 2 + 1 = -1 < 0$$

$$f(0) = 1 > 0$$

By intermediate value propety (note that f is continuous on $[0,1]$),

there exists $0 < x_0 < 1$, such that $f(x_0) = 0$, hence x_0 is a solution.

Bonus Problem. A toy rocket rises vertically in such a way that t seconds after liftoff, it is $h(t) = -16t^2 + 200t$ feet above the ground.

a. How high is the rocket after 6 seconds?

b. What is the average velocity of the rocket over the first 6 seconds?

c. What is the velocity of the rocket at liftoff? What is its velocity after 6 seconds?

a. $h(6) = -16 \cdot 36 + 200 \cdot 6$

$= -576 + 1200$

$= 624$

$$\begin{array}{r} 36 \\ 16 \\ \hline 216 \\ 36 \\ \hline 576 \end{array}$$

b. $\dfrac{h(6) - h(0)}{6} = \dfrac{624 - 0}{6} = 104$

c. $h'(t) = -32t + 200$

$h'(0) = 200$

$h'(6) = -32 \cdot 6 + 200$

$= -192 + 200$

$= 8.$