Name: ______________________________ Math 211 Quiz 4

Section: 302 ☐ 303 ☐ Feb 15, 2012

**1.** Compute the derivative of the given function.

$$\textbf{a. } f(t) = \frac{2}{\sqrt{t}} \quad \textbf{b. } g(x) = x^2(\frac{x^2}{2} + x + 1) \quad \textbf{c. } h(u) = \frac{1-u}{1+u}$$

a. $(2t^{-\frac{1}{2}})' = 2(t^{-\frac{1}{2}})' = 2\cdot(-\frac{1}{2})t^{-\frac{3}{2}} = \boxed{-\frac{1}{t^{3/2}}}$

b. $\left[(x^2)(\frac{x^2}{2}+x+1)\right]' = (x^2)'(\frac{x^2}{2}+x+1) + x^2\cdot(\frac{x^2}{2}+x+1)'$

$= 2x(\frac{x^2}{2}+x+1) + x^2(x+1)$

$= (x^3+2x^2+2x) + x^3 + x^2$

$= \boxed{2x^3+3x^2+2x}$

c. $\left(\frac{1-u}{1+u}\right)' = \frac{(1-u)'(1+u) - (1-u)(1+u)'}{(1+u)^2} = \frac{-(1+u)-(1-u)}{(1+u)^2} = \boxed{\frac{-2}{(1+u)^2}}$

**2.** Use the *definition* of derivative to find the derivative of the given function.

$$\textbf{a. } f(x) = x + 1 \quad \textbf{b. } g(t) = t^3$$

a. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h+1)-(x+1)}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = \boxed{1}$

b. $g'(t) = \lim_{h\to 0} \frac{g(t+h)-g(t)}{h} = \lim_{h\to 0} \frac{(t+h)^3 - t^3}{h}$

$= \lim_{h\to 0} \frac{(\cancel{t^3} + 3t^2h + 3th^2 + h^3) - \cancel{t^3}}{h}$

$= \lim_{h\to 0} \frac{3t^2\cancel{h} + 3th^{\cancel{2}} + h^{\cancel{3}\,2}}{\cancel{h}}$

$= \lim_{h\to 0} 3t^2 + 3th + h^2$

$= \boxed{3t^2}$

**3.** Compute the second derivative of the given function.

$$\text{a. } f(x) = ax^2 + bx + c \qquad \text{b. } g(t) = \frac{1}{t}$$

a. $f'(x) = 2ax + b$, $f''(x) = \boxed{2a}$

b. $g'(t) = (t^{-1})' = (-1)t^{-2}$, $g''(t) = (-1)(-2)t^{-3} = \boxed{2t^{-3}}$

$= \frac{2}{t^3}$

**Bonus problem.** Find numbers $a$, $b$,and $c$ such that the graph of the function $f(x) = ax^2 + bx + c$ will have $x$-intercepts at $(0, 0)$ and $(5, 0)$, and a tangent with slope 1 when $x = 2$.

$x$-intercept at $(0,0) \Rightarrow f(0) = 0 \Rightarrow c = f(0) = a0^2 + b0 + c = 0$

$x$-intercept at $(5,0) \Rightarrow f(5) = 0 \Rightarrow f(5) = 25a + 5b = 0$

$\Rightarrow 5a + b = 0$

tangent slope $= 1$ when $x = 2$ $\Rightarrow f'(2) = 1 \Rightarrow f'(2) = 2a2 + b = 1$

$\Rightarrow 4a + b = 1$

$$\begin{cases} 5a + b = 0 & \cdots ① \\ 4a + b = 1 & \cdots ② \end{cases}$$

① - ② $\Rightarrow (5a + b) - (4a + b) = 0 - 1$

$\Rightarrow a = -1$

$\Rightarrow b = 5$

conclusion: $\boxed{a = -1, \ b = 5, \ c = 0}$