

Name: _____

Math 211 Quiz 5

Section: 302

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Feb 22, 2012

Calculators are not allowed in this quiz.

1. Compute the derivative of the given function.

a. $f(t) = (t^3 - 1)(t - \frac{1}{t})$ b. $g(x) = \frac{x^2 + 1}{x + 1}$

a. $3t^2(t - \frac{1}{t}) + (t^3 - 1)(1 + \frac{1}{t^2})$
 $= 3t^3 - 3t + t^3 + t - 1 - \frac{1}{t^2}$
 $= 4t^3 - 2t - 1 - \frac{1}{t^2}$

b. $\frac{2x(x+1) - (x^2+1)}{(x+1)^2}$
 $= \frac{2x^2 + 2x - x^2 - 1}{(x+1)^2}$
 $= \frac{x^2 + 2x - 1}{(x+1)^2}$

2. Use the chain rule to compute the derivative $\frac{dy}{dx}$ and simplify the answer.

a. $y = \sqrt{u}; u = x^2 + 2x - 3$ b. $y = \frac{1}{u^2}; u = x^2 + 1$ c. $y = u^2 + u - 2; u = \frac{1}{x}$

a. $y' = \frac{1}{2} u^{-\frac{1}{2}} (2x+2)$
 $= \frac{1}{\sqrt{x^2+2x-3}} (2x+2)$
 $= \frac{x+1}{\sqrt{x^2+2x-3}}$

c. $y' = (2u+1) \cdot (-\frac{1}{x^2})$
 $= (\frac{2}{x} + 1) (-\frac{1}{x^2})$
 $= -\frac{2}{x^3} - \frac{1}{x^2}$

b. $y' = -2u^{-3} \cdot 2x$
 $= -2(x^2+1)^{-3} 2x$
 $= \frac{-4x}{(x^2+1)^3}$

3. Differentiate the given function and simplify your answer.

a. $f(x) = (x^2 + 1)^4$ b. $g(t) = \sqrt{t^2 + 1}$

a. $f'(x) = 4(x^2 + 1)^3(2x)$
 $= 8(x^2 + 1)^3x$

b. $g'(t) = \frac{1}{2} \frac{1}{\sqrt{t^2 + 1}} \cdot 2t$
 $= \frac{t}{\sqrt{t^2 + 1}}$

Bonus problem. The gross annual earnings of a certain company are $f(t) = \sqrt{10t^2 + t + 229}$ thousand dollars t years after its formation in January 2005.

a. At what rate will the gross annual earnings of the company be growing in January 2010?

b. At what percentage rate will the gross annual earnings be growing in January 2010?

a. $f'(t) = \frac{1}{2\sqrt{10t^2 + t + 229}} \cdot (20t + 1)$
 $= \frac{20t + 1}{2\sqrt{10t^2 + t + 229}}$

$f'(5) = \frac{20 \cdot 5 + 1}{2\sqrt{10 \cdot 25 + 5 + 229}} = \frac{101}{2\sqrt{484}} = \frac{101}{4\sqrt{121}} = \frac{101}{4 \cdot 11} = \frac{101}{44}$

b. percentage rate after 5 years $= \frac{100 f'(5)}{f(5)} = \frac{100 \frac{101}{44}}{22} = \frac{10100}{44 \cdot 22} = \frac{10100}{968} = 2525/242$