

Name: _____

Math 211 Quiz 6

Section: 302 303

Mar 7, 2012

Calculators are not allowed in this quiz.

1. Find the second derivative of the given function and simplify your answer.

a. $f(x) = (3x + 1)^5$ b. $f(x) = \sqrt{1 + x^2}$

a. $f'(x) = 5(3x+1)^4 \cdot 3$
 $= 15(3x+1)^4$

$f''(x) = 15 \cdot 4(3x+1)^3 \cdot 3$
 $= \boxed{180(3x+1)^3}$

b. $f'(x) = \frac{2x}{2\sqrt{1+x^2}}$

$= \frac{x}{\sqrt{1+x^2}}$

$f''(x) = \frac{\sqrt{1+x^2} - x \cdot \frac{x}{\sqrt{1+x^2}}}{\sqrt{1+x^2}^2}$

$= \frac{(\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}) \sqrt{1+x^2}}{(1+x^2) \sqrt{1+x^2}}$

$= \frac{(1+x^2) - x^2}{(1+x^2)^{3/2}}$

$= \boxed{\frac{1}{(1+x^2)^{3/2}}}$

2. Estimate how much the function $f(x) = x^2 - 3x + 5$ will change as x increases from 5 to 5.3.

$\Delta f = f(5.3) - f(5)$

$\approx f'(5) \Delta x$

$= f'(5)(5.3 - 5)$

$= f'(5) \cdot 0.3$

$= 7 \cdot 0.3$

$= \boxed{2.1}$

$\left(\begin{array}{l} f'(x) = 2x - 3 \\ f'(5) = 10 - 3 = 7 \end{array} \right)$

3. Find $\frac{dy}{dx}$ by implicit differentiation and simplify your answer.

a. $x^3 - y^2 = 5$ b. $(2x + y)^3 = x$

a. $3x^2 - 2yy' = 0$

$$3x^2 = 2yy'$$

$$y' = \frac{3x^2}{2y}$$

b. $3(2x+y)^2(2+y') = 1$

$$2+y' = \frac{1}{3(2x+y)^2}$$

$$y' = \frac{1}{3(2x+y)^2} - 2$$

Bonus problem. Use implicit differentiation to find the second derivative $\frac{d^2y}{dx^2}$.

$$x^2 + y^2 = 1$$

$$2x + 2yy' = 0$$

$$2yy' = -2x$$

$$y' = \frac{-2x}{2y}$$

$$= \frac{-x}{y}$$

$$y'' = \left(\frac{-x}{y}\right)'$$

$$= \frac{(-1)y - (-x)y'}{y^2}$$

$$= \frac{-y + xy'}{y^2}$$

$$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$= \frac{-y - \frac{x^2}{y}}{y^2} = \frac{(y)}{(y)}$$

$$= \frac{-y^2 - x^2}{y^3} = \boxed{\frac{-1}{y^3}} \quad (x^2 + y^2 = 1)$$