Name: ______________________________ Math 211 Quiz 7

Section: 302 ☐ 303 ☐ Mar 14, 2012

Calculators are not allowed in this quiz.

1. Determine the critical points of the given function and classify each critical point as a relative maximum, a relative minimum, or neither.

$$f(t) = \frac{t}{t^2+3}$$

$$f'(t) = \frac{(t^2+3) - t(2t)}{(t^2+3)^2}$$

$$= \frac{-t^2+3}{(t^2+3)^2}$$

$f'(t) = 0 \Rightarrow -t^2+3 = 0$

$\Rightarrow t = \pm\sqrt{3}$

$\Rightarrow \boxed{\left(-\sqrt{3}, \frac{-\sqrt{3}}{6}\right)}$ and $\boxed{\left(\sqrt{3}, \frac{\sqrt{3}}{6}\right)}$ are the critical points.

f: ↘ ↗ ↘

f': $---$ $+++$ $---$

$-\sqrt{3}$ $\sqrt{3}$

$\left(-\sqrt{3}, \frac{-\sqrt{3}}{6}\right)$ is a relative min.

$\left(\sqrt{3}, \frac{\sqrt{3}}{6}\right)$ is a relative max.

2. Determine where the given function is concave up and concave down. Find the inflection point and use the second derivative test to find the extrema.

$$f(x) = \frac{1}{3}x^3 - 9x + 2$$

$f'(x) = x^2 - 9$ ($\Rightarrow x = \pm 3$ are the critical points)

$f''(x) = 2x$

f: ↗ ↘ ↗

f': $+++$ $---$ $+++$

-3 0 3

f'': $---$ $+++$

f: ⌒ ⌣

$f''(x)$ changes sign at $x=0 \Rightarrow (0, f(0)) = \boxed{(0,2)}$ is an inflection point.

$f'(-3) = 0,\ f''(-3) < 0 \Rightarrow (-3, f(-3)) = \boxed{(-3, 20)}$ is a relative max

$f'(3) = 0,\ f''(3) > 0 \Rightarrow (3, f(3)) = \boxed{(3, -16)}$ is a relative min.

Bonus problem. An efficiency study of the morning shift (from 8:00 A.M. to 12:00 noon) at a factory indicates that an average worker who arrives on the job at 8:00 A.M. will have produced Q units t hours later, where

$$Q(t) = -t^3 + \frac{9}{2}t^2 + 15t.$$

a. At what time during the morning is the worker performing most efficiently?
b. At what time during the morning is the worker performing least efficiently?

The domain of $Q(t)$ is $[0, 4]$ because we are studying the morning shift (from 8 to 12)

The efficiency of an average work is $Q'(t) = -3t^2 + 9t + 15$

a. Maximize $Q'(t)$ on $[0, 4]$:

- Solve $Q''(t) = 0$ for the critical points of $Q'(t)$, i.e. $-6t + 9 = 0$, $t = \frac{3}{2}$.
- Plug in the critical pt and end pts to Q', we get $Q'(\frac{3}{2}) = \frac{87}{4}$, $Q'(0) = 15$; $Q'(4) = 3$
- $\max_{0 \le t \le 4} Q'(t) = \max\{\frac{87}{4}, 15, 3\} = \frac{87}{4}$.

So at $\boxed{9{:}30\text{ AM}}$ ($\frac{3}{2}$ hours after 8:00 AM) the worker is working most efficiently.

b. Minimize $Q'(t)$ on $[0,4]$:

- $\min_{0 \le t \le 4} Q'(t) = \min\{\frac{87}{4}, 15, 3\} = 3$

So at 12:00 noon (4 hours after 8:00 AM) the worker is working least efficiently.