Name: ______________________________ Math 211 Quiz 8

Section: 302 ☐ 303 ☐ Mar 21, 2012

1. Find the absolute maximum and absolute minimum of the given function on the specified interval.

$$f(x) = x^2 + 4x + 5; -3 \le x \le 1$$

$f'(x) = 2x + 4 = 0 \Rightarrow x = -2$ (critical pt)

$f(-2) = (-2)^2 - 8 + 5 = 1$

$f(-3) = (-3)^2 - 12 + 5 = 2$

$f(1) = 1 + 4 + 5 = 10$

$\max\limits_{-3 \le x \le 1} f = \max\{1, 2, 10\} = 10$

$\min\limits_{-3 \le x \le 1} f = \min\{1, 2, 10\} = 1$

2. A Florida citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

x — # of trees , $x \ge 0$

$f(x)$ — yield total

$a(x)$ — average yield (per tree)

$f(x) = x\, a(x)$

$a(x) = 400 - 4(x - 60)$

$= -4x + 640$

So $f(x) = x(-4x + 640)$

$= -4x^2 + 640x$

Want to maximize $f(x)$ on $[0, \infty)$.

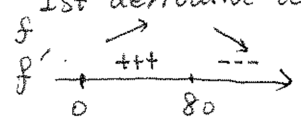
$f'(x) = -8x + 640 = 0$

$\Rightarrow x = 80$ (only critical pt)

$f''(x) = -8$, $f''(80) < 0$

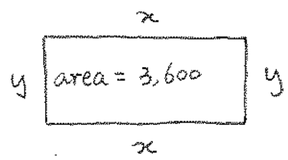
$\Rightarrow f(80)$ is absolute max (by 2nd derivative test)

one can also use 1st derivative test:



Conclusion: the grower should plant $\boxed{80}$ trees to maximize the total yield.

Bonus problem. A city recreation department plans to build a rectangular playground having an area of 3,600 square meters and surround it by a fence. How can this be done using the least amount of fencing?



$$\text{area} = xy = 3600 \Rightarrow y = \frac{3600}{x}.$$

$$\underset{\substack{\| \\ f(x)}}{\text{fencing}} = 2x + 2y = 2x + 2\cdot\frac{3600}{x}, \quad x > 0$$

Want to minimize $f(x)$ on $(0, \infty)$.

$$f'(x) = 2 - \frac{7200}{x^2}$$

$$f'(x) = 0 \Rightarrow 2 = \frac{7200}{x^2}$$

$$\Rightarrow x^2 = 3600 \Rightarrow x = 60$$

(the only critical pt)

again, one can also use the 1st derivative test

$$f''(x) = \frac{2\cdot 7200}{x^3}$$

$f''(60) > 0 \Rightarrow f(60)$ is absolute minimum

(by the 2nd derivative test)

Conclusion: $\boxed{60 \times 60}$ playground uses the least amount of fencing.