

Math 213 Midterm 1 Review

Calculator is not allowed.

1. Find the derivatives of the following functions.

- (a) $f(x) = (2x^2 + 3x + 1)^2 + x^2$
- (b) $f(x) = \sqrt{4x^2 + 1}$
- (c) $f(x) = \frac{2}{\pi^2 \sqrt{1-x^2}}$
- (d) $f(x) = \frac{e^{-2x}}{2x} - \frac{e}{x}$
- (e) $f(x) = x^2(\ln x) - \frac{x^2}{2}$.
- (f) Compute $f'(1)$ and $f'(e)$ where $f(x)$ is as in (e).

2. Find the local maximum and minimum of the function $f(x) = x^3 - 3x + 1$.

3. Find the following integrals.

- (a) $\int (x + 1)^2 + x + 1 dx$
- (b) $\int_0^4 \frac{2x}{\sqrt{x^2+9}} - x\sqrt{x^2+9} dx$
- (c) $\int x^2 e^{x^3} + 2 dx$
- (d) $\int_e^{e^2} \frac{1}{x \ln x} dx$
- (e) $\int_0^1 x e^{2x} - x dx$
- (f) $\int_1^2 x^2 \ln x dx$.

3. Use the trapezoidal rule and Simpson's rule to approximate the value of the definite integral for $n = 4$.

$$\int_0^4 x^2 dx.$$

4. Let $A = \langle 1, 3, 2 \rangle$, $B = \langle 2, 5, 4 \rangle$, $C = \langle -1, 2, 4 \rangle$ be three points in the space.

- (a) Find the vectors \overrightarrow{AB} and \overrightarrow{AC} .
- (b) Find the distances between A and B and between A and C .
- (c) Use inner product to find the angle between \overrightarrow{AB} and \overrightarrow{AC} .
- (d) Verify that

$$\|\overrightarrow{AB} + \overrightarrow{AC}\|^2 + \|\overrightarrow{AB} - \overrightarrow{AC}\|^2 = 2\|\overrightarrow{AB}\|^2 + 2\|\overrightarrow{AC}\|^2.$$

5. (a) Find the equation of the plane perpendicular to $n = (-1, 1, 2)$ and going through the point $P = (2, -2, -4)$. (b) Find the intercepts of the plane with the axes.

6. Give examples of equations describing the following types of surfaces: (a) ellipsoid (b) elliptic cone (c) elliptic paraboloid (d) hyperbolic paraboloid (e) hyperboloid of one sheet (f) hyperboloid of two sheets.

7. Let $f(x, y) = e^{-x^2-y^2}$ and $g(x, y) = \frac{xy}{x^2+y^2}$.

- (a) Find f_{xy} and f_{xx} .
- (b) Find $g_y(1, 1)$.