

Math 213 Final Exam Review - Part 2

1. Find the minimum value of the function $z = x^2 + y^2 + xy - 2x + 2y$.
2. Find all critical points of the function $z = \ln(1 + x^2 + y^2)$ and determine their type.
3. Let $S = yz + xz + xy$. Use Lagrange multipliers to minimize S under the constraint $xyz = 1000$.
4. Find the line of best fit to the data points $\{(-1, 1), (0, 0), (2, 0)\}$.
5. Find the general solution to the differential equation

$$xy' = \frac{y^2}{x^2}.$$

6. Find the general solution to the differential equation

$$y' = xy + x.$$

7. Find the particular solution with initial condition $y(1) = 0$ to the differential equation

$$y' + \frac{1}{x}y = e^{-x^2}.$$

8. Three coins are tossed.
 - a) What is the sample space?
 - b) What is the event that there are exactly two heads?
 - c) What is the probability of getting two heads?
 - d) Let the number of heads be a random variable x . What is the expected value of x ?
 - e) What is the variance of x ?
9. A random variable x has probability density function

$$f(x) = \frac{3}{4}x(2 - x), \quad 0 \leq x \leq 2.$$

- a) What is the probability that $x \leq 1$?
 - b) What is the mean of x ?
 - c) What is the variance of x ?
 - d) What is the standard deviation of x ?
10. A random variable x has probability density function

$$f(x) = ke^{-x}, \quad 0 \leq x \leq 1.$$

- a) What should the constant k be?
- b) What is the mean of x ?

c) What is the variance of x ?

11. Classify the following functions as one of the following: paraboloid, ellipsoid, hyperbolic paraboloid, hyperboloid of one sheet, hyperboloid of two sheets, or none of the above.

a) $z = x^2 - y^2$

b) $z = x^2 + y^2$

c) $z^2 = x^2 + 3y^2$

d) $z^2 + x^2 + 4y^2 = 1$

e) $z^2 + x^2 - 5y^2 = 1$

f) $z^2 - x^2 - 6y^2 = 1$

12. Consider the differential equation

$$y' = y^2 + x$$

with initial condition $y(0) = 1$. Use Euler's method to estimate $y(2)$ using step size 1 and using step size $1/2$.

13. The rate of decomposition of radioactive carbon is proportional to the amount present at any time. The half-life of radioactive carbon is 5715 year. What percent of a present amount will remain after 1000 years?