Math 213 Discussion Worksheet – Week 5

Starred problems are optional

1. Indicate the types (ellipsoid/elliptic cone/elliptic paraboloid/hyperbolic paraboloid/ hyperboloid of one sheet/hyperboloid of two sheets) of the following equations.
(a) $x^2 - y^2 - z^2 = 1$ (b) $x^2 + y^2 - z^2 = 1$ (c) $x^2 + y^2 - z^2 = 0$ (d) $x - y^2 - z^2 = 0$ (e) $x - y^2 + z^2 = 0$.

Solution. (a) hyperboloid of two sheets (b) hyperboloid of one sheet (c) elliptic cone (d) elliptic paraboloid (e) hyperbolic paraboloid.

2. Describe the traces of $z^2 = x^2 + y^2$ in the given planes.
(a) $y = 1$ (b) $z = 1$ (c) $x = 0$ (d)* $z - y = 1$ (e)* $z - y = 0$ (f)* $2z - y = 1$.

Solution.
(a) $z^2 - x^2 = 1$ (a hyperbola)
(b) $x^2 + y^2 = 1$ (a circle)
(c) $z^2 = y^2$ (i.e. $z = y$ or $z = -y$, two lines)
(d) a parabola
(e) a line
(f) an ellipse.

3. Sketch a contour map of the surface using level curves for the given c-values.
(a) $z = 2 - 2x - y, c = -2, 0, 2, 4$
(b) $z = e^{-x^2 - y^2}, c = 1, e^{-1}, e^{-4}$.

Solution.
(a) 4 parallel lines (picture omitted)
(b) 3 concentric circles centered at 0 (picture omitted).

4. Let $f(x, y) = xye^{-x-y}$ and $g(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$.
(a) Find $f_{xx}(x, y)$ and $f_{xy}(x, y)$
(b) Find $g_x(1, 0)$ and $g_y(1, 0)$.

Solution.
(a)
$$f_{xx}(x, y) = (-2y + xy)e^{-x-y} = (x - 2)ye^{-x-y},$$
$$f_{xy}(x, y) = (1 - x - y + xy)e^{-x-y} = (1 - x)(1 - y)e^{-x-y}.$$
(b)
$$g_x(x, y) = \frac{y^2}{(x^2 + y^2)^{3/2}} \text{ (after simplification)},$$
$$g_y(x, y) = -\frac{xy}{(x^2 + y^2)^{3/2}} \text{ (after simplification)}.$$
Hence $g_x(1, 0) = 0$, $g_y(1, 0) = 0$.