HW 2 # 4. (a) $\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{2t}{3} \\ \frac{t^2}{3} \\ 0 \end{pmatrix}$ (c) parabola

HW2 # 5. $L = 2\pi\sqrt{R^2+a^2}$

$P = 2\pi R$

$H = 2\pi a$

$L^2 = P^2 + H^2$ ✓

HW2#7. (a) $k(t) = \dfrac{2}{(1+4t^2)^{3/2}}$, maximized at $t=0$;

(b) $k(t) = \dfrac{6}{|t|(4+9t^2)^{3/2}}$, maximized at $t=0$

(c) $k(t) = \dfrac{\sqrt{1+a^2}}{(R^2+a^2)^{3/2}}$, constant.

(d) see the .pdf text.

<u>Solution for (b)</u>:

1) curvature $k = \dfrac{1}{\|\vec{x}'\|^3}\|\vec{x}' \times \vec{x}''\|$.

$$\vec{x}' = \begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix}, \quad \vec{x}'' = \begin{pmatrix} 2 \\ 6t \\ 0 \end{pmatrix}, \quad \vec{x}' \times \vec{x}'' = \begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 6t \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6t^2 \end{pmatrix}$$

$$\|\vec{x}' \times \vec{x}''\| = 6t^2, \quad \|\vec{x}'\| = \sqrt{4t^2+9t^4} = |t|(4+9t^2)^{1/2}$$

So, $$k = \frac{6t^2}{\left[|t|(4+9t^2)^{1/2}\right]^3} = \frac{6t^2}{|t|^3(4+9t^2)^{3/2}} = \boxed{\frac{6}{|t|(4+9t^2)^{3/2}}}.$$

2) Since both $|t|$ and $(4+9t^2)$ are minimized at $t=0$

$k(t)$ is maximized at $t=0$, and $\boxed{k(0)=\infty}$.

3) $$\boxed{\vec{T}} = \frac{\vec{x}'}{\|\vec{x}'\|} = \frac{\begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix}}{\left\|\begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix}\right\|} = \boxed{\frac{1}{|t|(4+9t^2)^{1/2}}\begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix}}.$$

4) $\boxed{\vec{N}} = \dfrac{\vec{\kappa}}{\|\vec{\kappa}\|}$.

$$\vec{\kappa} = \frac{1}{\|\vec{x}'\|} \frac{d}{dt}\left\{ \frac{1}{\|\vec{x}'\|} \vec{x}' \right\}$$

$$= \frac{1}{|t|\sqrt{4+9t^2}} \frac{d}{dt}\left\{ \frac{1}{\sqrt{4t^2+9t^4}} \begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix} \right\}$$

(if $t>0$)

$$= \frac{1}{|t|\sqrt{4+9t^2}} \frac{d}{dt}\left\{ \frac{1}{\cancel{t}\sqrt{4+9t^2}} \cdot \cancel{t} \begin{pmatrix} 2 \\ 3t \\ 0 \end{pmatrix} \right\}$$

$$= \frac{1}{t\sqrt{4+9t^2}} \left\{ \frac{-1}{2}(4+9t^2)^{-\frac{3}{2}}(18t)\begin{pmatrix} 2 \\ 3t \\ 0 \end{pmatrix} + (4+9t^2)^{-\frac{1}{2}} \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \right\}$$

$$= \frac{1}{t\sqrt{4+9t^2}} \left\{ -(4+9t^2)^{-\frac{3}{2}} 9t \begin{pmatrix} 2 \\ 3t \\ 0 \end{pmatrix} + (4+9t^2)^{-\frac{3}{2}} (4+9t^2)^{1} \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} \right\}$$

$$= \frac{1}{t\sqrt{4+9t^2}} (4+9t^2)^{-\frac{3}{2}} \left\{ \begin{pmatrix} -18t \\ -27t^2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3(4+9t^2) \\ 0 \end{pmatrix} \right\}$$

$$= \frac{1}{t(4+9t^2)^2} \begin{pmatrix} -18t \\ 12 \\ 0 \end{pmatrix}$$

$$= \underbrace{\frac{6}{t(4+9t^2)^2}}_{c} \underbrace{\begin{pmatrix} -3t \\ 2 \\ 0 \end{pmatrix}}_{\vec{u}}$$

$$\vec{N} = \frac{\vec{\kappa}}{\|\vec{\kappa}\|} = \frac{c\vec{u}}{\|c\vec{u}\|} = \frac{\cancel{c}\vec{u}}{\cancel{c}\|\vec{u}\|} = \frac{\begin{pmatrix} -3t \\ 2 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} -3t \\ 2 \\ 0 \end{pmatrix} \right\|} = \boxed{\frac{1}{\sqrt{9t^2+4}} \begin{pmatrix} -3t \\ 2 \\ 0 \end{pmatrix}}$$

5) $\boxed{\vec{B}} = \vec{T} \times \vec{N}$

$$= \left[\frac{1}{t(4+9t^2)^{1/2}} \begin{pmatrix} 2t \\ 3t^2 \\ 0 \end{pmatrix} \right] \times \left[\frac{1}{(4+9t^2)^{1/2}} \begin{pmatrix} -3t \\ 2 \\ 0 \end{pmatrix} \right] = \frac{1}{t(4+9t^2)} \begin{pmatrix} 0 \\ 0 \\ 4t+9t^3 \end{pmatrix} = \boxed{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$$