Name: ______________________ Math 234 Quiz 10

Section: 328 ☐ 329 ☐ Dec 4, 2014

1. Let $\mathcal{C}$ be the counter-clockwise traversed boundary of the region $\mathcal{R}$. Compute the indicated line integral using Green's theorem.

(a) (10 pts) $\oint_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} ds$, $\mathcal{R} : x^2 + y^2 \leq 4$ and $y \geq 0$

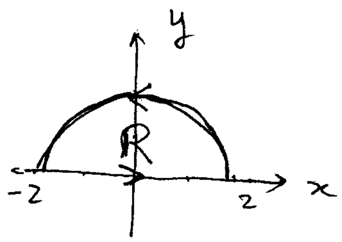
where $\overrightarrow{\mathbf{F}} = (xy^2, x^2y)$ and $\overrightarrow{\mathbf{N}}$ is the outward normal.

(b) (10 pts) $\oint_C (x-y)dx + (x+y)dy$, $\mathcal{R} : 0 \leq x \leq 1, 0 \leq y \leq x^2$.

Bonus. (5 pts) Compute the integral in **1**(b) without using Green's theorem.

1(a). By Green's thm,

$$\oint_C \vec{F}\cdot\vec{N}\,ds = \iint_R (y^2+x^2)\,dA = \int_0^{\pi}\int_0^{2} r^2 \cdot r\,dr\,d\theta = \int_0^{\pi} \frac{r^4}{4}\Big|_0^2 d\theta = \boxed{4\pi}$$



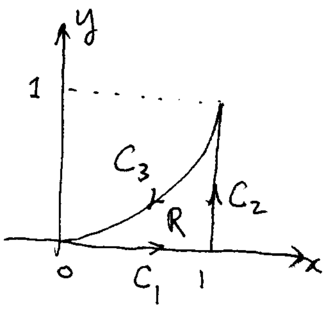
1(b). By Green's thm,

$$\oint_C (x-y)\,dx + (x+y)\,dy$$

$$= \iint_R \left[\frac{\partial}{\partial x}(x+y) - \frac{\partial}{\partial y}(x-y)\right] dA$$

$$= \iint_R 2\,dA = \int_0^1\int_0^{x^2} 2\,dy\,dA = \int_0^1 2x^2\,dx = \boxed{\frac{2}{3}}$$

Bonus.



$$\oint_C = \int_{C_1} + \int_{C_2} + \int_{C_3} = \frac{1}{2} + \frac{3}{2} - \frac{4}{3} = \boxed{\frac{2}{3}}$$

$$\int_{C_1} = \int_0^1 (x-0)\,dx + (x+0)\cancel{d0}^{\,0} = \int_0^1 x\,dx = \frac{1}{2}$$

$$\int_{C_2} = \int_0^1 (1-y)\cancel{d0}^{\,0} + (1+y)\,dy = \int_0^1 (1+y)\,dy = \frac{3}{2}$$

$$\int_{C_3} = -\left(\int_0^1 (x-x^2)\,dx + (x+x^2)\underbrace{d(x^2)}_{2x\,dx}\right)$$

$$= -\int_0^1 x\,dx + \int_0^1 x^2\,dx - \int_0^1 2x^2\,dx - \int_0^1 2x^3\,dx$$

$$= -\frac{1}{2} + \frac{1}{3} - \frac{2}{3} - \frac{2}{4} = -1 - \frac{1}{3} = -\frac{4}{3}$$