Name: ______________________________ Math 234 Quiz 1

Section: 328 ☐ 329 ☐ Sep. 2014

1. (10 pts) Let

$$\vec{\mathbf{a}} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \vec{\mathbf{c}} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Compute the triple product $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$.

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0\cdot\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 1\cdot\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1\cdot\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 0\cdot(-1) - 1\cdot(-1) + 1\cdot 1$$

$$= 0 + 1 + 1$$

$$= \boxed{2}$$

2. (10 pts) Let $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be as above. Use dot product to find the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$. (*Hint:* $\cos(60°) = 1/2$.)

$$\vec{a}\cdot\vec{b} = \|\vec{a}\|\,\|\vec{b}\|\cos\theta \quad (\theta = \angle(\vec{a},\vec{b}))$$

$$\Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{0^2+1^2+1^2}\cdot\sqrt{1^2+0^2+1^2}}$$

$$= \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \boxed{60°} = \boxed{\frac{\pi}{3}}$$

Bonus. (5 pts) Let $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ be two vectors (not necessarily the same as above). Use dot product to prove the *parallelogram law*:

$$\|\vec{\mathbf{a}} + \vec{\mathbf{b}}\|^2 + \|\vec{\mathbf{a}} - \vec{\mathbf{b}}\|^2 = 2(\|\vec{\mathbf{a}}\|^2 + \|\vec{\mathbf{b}}\|^2).$$

$$\|\vec{a}+\vec{b}\|^2 + \|\vec{a}-\vec{b}\|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) + (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$= (\vec{a}\cdot\vec{a} + \cancel{2\vec{a}\cdot\vec{b}} + \vec{b}\cdot\vec{b}) + (\vec{a}\cdot\vec{a} - \cancel{2\vec{a}\cdot\vec{b}} + \vec{b}\cdot\vec{b})$$

$$= 2\|\vec{a}\|^2 + 2\|\vec{b}\|^2$$

$$= 2(\|\vec{a}\|^2 + \|\vec{b}\|^2).$$