

Name: _____

Math 234 Quiz 3

Section: 328 329

Sep 23, 2014

1. (10 pts) (a) Find the equation of the plane that contains the points $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 1, -1)$. (b) Find the intersection of the plane with the z -axis.

(a) Suppose $z = ax + by + c$,

$$\text{then } \begin{cases} 0 = a + c & (1) \\ 0 = b + c & (2) \\ -1 = a + b + c & (3) \end{cases}$$

$$(2) - (1) \Rightarrow b - c = 0 \Rightarrow b = c$$

$$(3) - (2) \Rightarrow a = -1$$

$$(3) - (1) \Rightarrow b = -1$$

$$(\Rightarrow c = b = -1)$$

$$\text{So, } \boxed{z = -x - y + 1}$$

(b) Set $x=0$ and $y=0$,

$$\text{get } z = -0 - 0 + 1$$

$$= \boxed{1}$$

So the intersection is

$$\boxed{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$$

2. (10 pts) Classify the quadratic form

$$Q(x, y) = -2x^2 + 4xy - 6y^2$$

as definite, indefinite, or semidefinite.

$$\Delta = 4AC - B^2 = 4(-2)(-6) - 4^2$$

$$= 48 - 16$$

$$= 32 > 0$$

So $Q(x, y)$ is (negative) definite.

Bonus. (5 pts) Let $Q(x, y)$ be as above. Rewrite $Q(x, y)$ in terms of polar coordinates (i.e. r and θ), and simplify the expression as much as possible.

$$Q(x, y) = Q(r \cos \theta, r \sin \theta)$$

$$= -2(r \cos \theta)^2 + 4(r \cos \theta)(r \sin \theta) - 6(r \sin \theta)^2$$

$$= -2r^2(\cos^2 \theta - 2 \cos \theta \sin \theta + 3 \sin^2 \theta)$$

$$= -2r^2(\cos^2 \theta - 2 \cos \theta \sin \theta + \sin^2 \theta + 2 \sin^2 \theta)$$

$$= -2r^2(1 - 2 \cos \theta \sin \theta + 2 \sin^2 \theta)$$

$$= \boxed{-2r^2(2 - \sin(2\theta) - \cos(2\theta))}$$