Name: _____ Math 234 Quiz 4

Section: 328 ☐ 329 ☐ Sep 30, 2014

1. Let $f(x,y) = e^{-x^2-y^2}$.
   (1) (5 pts) Compute the gradient of f at $(1,1)$.
   (2) (2 pts) In which direction does f decrease fastest at $(1,1)$?
   (3) (5 pts) Find the linear approximation of f at $(1,1)$.
   (4) (3 pts) Use (3) to estimate $f(1.1, 1.1)$. (Note that $e^{-2} \approx 0.14$).
   (5) (5 pts) Find the equation of the tangent plane to the graph of f at $(1,1)$.
   (6) (3 pts) Find the equation of the tangent line to the level curve of f at $(1,1)$.
   (7) (2 pts) Sketch the graph of f.
   The last two questions are for bonus.

(1) $\dfrac{\partial f}{\partial x} = -2x\, e^{-x^2-y^2}$

$\dfrac{\partial f}{\partial y} = -2y\, e^{-x^2-y^2}$

$\Rightarrow \nabla f(1,1) = \boxed{(-2e^{-2}, -2e^{-2})}$

(2) $-\nabla f(1,1) = (2e^{-2}, 2e^{-2})$

(3) $f(x,y) \approx f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \boxed{e^{-2} - 2e^{-2}(x-1) - 2e^{-2}(y-1)}$

(4) $f(1.1, 1.1) \approx e^{-2} - 2e^{-2}(0.1) - 2e^{-2}(0.1)$

$= (0.6)\, e^{-2}$

$\approx (0.6)(0.14)$

$= \boxed{0.084}$

(5) $z = e^{-2} - 2e^{-2}(x-1) - 2e^{-2}(y-1)$

(6) $0 = -2e^{-2}(x-1) - 2e^{-2}(y-1)$

$\boxed{y = -x + 2}$

(7)