Name: ______________________ Math 234 Quiz 5

Section: 328 ☐ 329 ☐ Oct 16, 2014

1. (10 pts) Assuming that the function

$$f(x, y) = x^2 + xy + y^2 - 3x - 3y - 3$$

has a global minimum. Find the value of the minimum.

$$\begin{cases} f_x = 2x + y - 3 = 0 \cdots (1) \\ f_y = x + 2y - 3 = 0 \cdots (2) \end{cases}$$

$(1) - (2)\times 2 \Rightarrow -3y + 3 = 0$

$\Rightarrow y = 1$

$\overset{(2)}{\Rightarrow} x = 1$

$\Rightarrow$ c.p.: $\boxed{(1,1)}$

Since $(1,1)$ is the unique c.p., it has to be the minimum point by the assumption.

The corresponding value is

$f(1,1) = 1+1+1-3-3-3 = \boxed{-6}$

2. (10 pts) Find the critical points of the function

$$f(x, y, z) = x^3 + y^3 + z^3 - 3x^2 - 3y - 3.$$

$$\begin{cases} f_x = 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x = 0, 2 \\ f_y = 3y^2 - 3 = 0 \Rightarrow 3(y^2 - 1) = 0 \Rightarrow y = \pm 1 \\ f_z = 3z^2 = 0 \Rightarrow z = 0 \end{cases}$$

So the c.p.'s are $\boxed{(0,1,0),\ (0,-1,0),\ (2,1,0),\ (2,-1,0)}$.

Bonus. (5 pts) Justify that the function $f(x, y)$ in Problem 1 has a global minimum.

$$\begin{cases} f_{xx}(1,1) = 2 \\ f_{xy}(1,1) = 1 \\ f_{yy}(1,1) = 2 \end{cases}$$

<u>2nd der. test</u>: $f_{xx} > 0$, $f_{xx}f_{yy} - (f_{xy})^2 = 3 > 0$

So $(1,1)$ is a <u>local min</u>.

Since $(1,1)$ is the <u>unique c.p.</u>, this local min is a fact a <u>global min</u>.