Name: ____________________ Math 234 Quiz 6

Section: 328 ☐ 329 ☐ Oct 23, 2014

1. Consider the function

$$f(x,y) = 2x^2 - 4xy + y^4.$$

(a) (10 pts) Find the critical points of f.

(b) (10 pts) Apply the second derivative test to the points of you find.

(a) $\begin{cases} f_x = 4x - 4y = 0 & \Rightarrow x = y \\ f_y = -4x + 4y^3 = 0 & \Rightarrow -4x + 4x^3 = 0 \end{cases}$

$4x(-1 + x^2) = 0$

$\Rightarrow x = 0, \pm 1$

So the critical pts are: $\boxed{(0,0), (1,1), (-1,-1)}$

(b) $f_{xx} = 4$, $f_{xy} = -4$, $f_{yy} = 12y^2$

@ $(0,0)$: $\begin{pmatrix} 4 & -4 \\ -4 & 0 \end{pmatrix}$, det $= -16 < 0$, so it is a $\boxed{\text{saddle pt.}}$

@ $(1,1)$: $\begin{pmatrix} 4 & -4 \\ -4 & 12 \end{pmatrix}$, det $= 4\cdot 12 - 4\cdot 4 > 0$, $f_{xx} = 4 > 0$, so it is a $\boxed{\text{loc min}}$.

@ $(-1,-1)$: $\begin{pmatrix} 4 & -4 \\ -4 & 12 \end{pmatrix}$, same as above, it is a $\boxed{\text{loc min}}$.

Bonus. (5 pts) Find the maximum and minimum (values) of the above function.

(1) $\boxed{\max f = \infty}$, Since $\lim_{y\to\infty} f(0,y) = \lim_{y\to\infty} y^4 = \infty$.

(2) $f(0,0) = 0$, $f(1,1) = -1$, $f(-1,-1) = -1$

Since $\lim_{\substack{|x|\to\infty \\ |y|\to\infty}} f(x,y) = \infty$, f has a global minimum, occuring

at one of the critical points, $(1,1)$ or $(-1,1)$. So $\boxed{\min f = -1}$.