Name: ______________________

Section: 328 ☐ 329 ☐

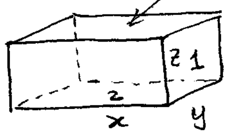
Math 234 Quiz 7

Oct 30, 2014

1. (a) (20 pts) The material for the bottom of an aquarium costs twice as much per unit area as the sides. Find the shape of the cheapest aquarium that has diagonal length 3.

(b) (5 pts, bonus) What if the material for the bottom has no cost?

(a)



Cost $= \overbrace{\underline{\underline{2}}xy}^{\text{bottom}} + 2yz + 2zx$

constraint: $\sqrt{x^2+y^2+z^2} = 3$

$\Leftrightarrow \quad x^2+y^2+z^2=9$

$$\begin{cases} f = 2xy+2yz+2zx \\ g = x^2+y^2+z^2-9 \end{cases}$$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (2y+2z = \lambda 2x) \ \underline{+\ 2x} \\ (2x+2z = \lambda 2y) \ \underline{+\ 2y} \\ (2x+2y = \lambda 2z) \ \underline{+\ 2z} \\ x^2+y^2+z^2-9=0 \end{cases}$$

$$\Rightarrow \begin{cases} 2x+2y+2z = \lambda 2x + 2x \\ 2x+2y+2z = \lambda 2y + 2y \\ 2x+2y+2z = \lambda 2z + 2z \\ x^2+y^2+z^2=9 \end{cases}$$

$\Rightarrow (2\lambda+2)x = (2\lambda+2)y \overset{\lambda>0}{\Rightarrow} x=y$

$\Rightarrow (2\lambda+2)y = (2\lambda+2)z \Rightarrow y=z$

$\Rightarrow \boxed{x=y=z}$

$\Rightarrow 3x^2 = 9 \Rightarrow x=\sqrt{3} \Rightarrow y=z=\boxed{\sqrt{3}}$

So the cheapest aquarium has shape $\sqrt{3}\times\sqrt{3}\times\sqrt{3}$.

(b)

$$\begin{cases} f = 2yz+2zx \\ g = x^2+y^2+z^2-9 \end{cases}$$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 2z = \lambda 2x \\ 2z = \lambda 2y \\ 2y+2x = \lambda 2z \end{cases}$$

$\Rightarrow 2\lambda x = 2\lambda y \Rightarrow \boxed{x=y}$

$\Rightarrow 4x = \lambda 2z = \lambda 2(\lambda x) = 2\lambda^2 x$

$\Rightarrow 4x = 2\lambda^2 x \Rightarrow 2=\lambda^2 \Rightarrow \lambda=\sqrt{2} \quad (\lambda>0)$

$\Rightarrow 4x = 2\sqrt{2}z \Rightarrow \boxed{z=\sqrt{2}x}$

$x^2+y^2+z^2=9 \Rightarrow 4x^2=9 \Rightarrow x=\frac{3}{2} \Rightarrow y=\frac{3}{2},\ z=\frac{3\sqrt{2}}{2}$

So the cheapest aquarium has shape $\boxed{\frac{3}{2}\times\frac{3}{2}\times\frac{3\sqrt{2}}{2}}$