Name: _____ Math 234 Quiz 8

Section: 328 ☐ 329 ☐ Nov 6, 2014

1. (20 pts) Compute the following double integrals.

(a) $\iint_D 2xe^y dA$, where $D = \{(x,y) : 0 \le x \le 1, 0 \le y \le x^2\}$.

(b) $\iint_D y^2 \cos(x^2) dA$, where $D = \{(x,y) : 0 \le y \le 100, y^3 \le x \le \sqrt{\pi}\}$

(a) $$\int_0^1 \int_0^{x^2} 2xe^y \, dy\, dx = \int_0^1 \left(2x e^y \Big|_{y=0}^{y=x^2} \right) dx$$

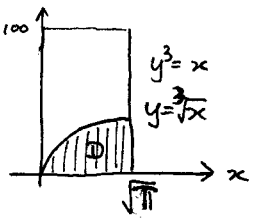
$$= \int_0^1 \left(2x e^{x^2} - 2x \right) dx$$

$$\overset{\substack{u=x^2 \\ du=2x\,dx}}{=} \int_0^1 e^u du - \int_0^1 2x\,dx$$

$$= e^u \Big|_0^1 - x^2 \Big|_0^1$$

$$= (e-1) - 1 = \boxed{e-2}$$

(b) $$\iint_D y^2 \cos(x^2)\, dA = \int_0^{\sqrt{\pi}} \int_0^{\sqrt[3]{x}} y^2 \cos(x^2)\, dy\, dx$$



$$= \int_0^{\sqrt{\pi}} \left(\cos(x^2) \frac{y^3}{3} \Big|_{y=0}^{y=\sqrt[3]{x}} \right) dx$$

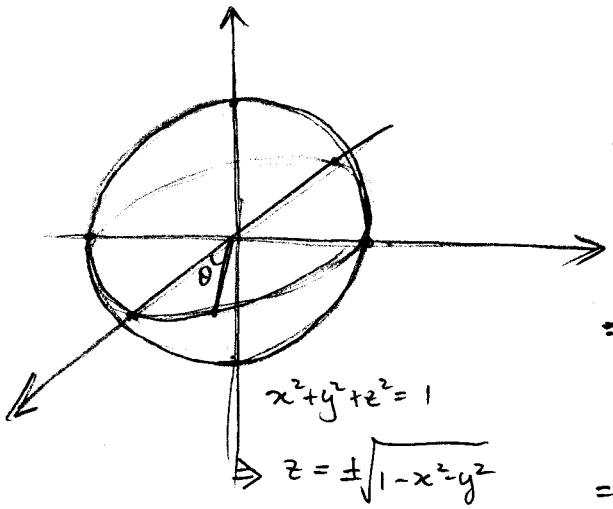
$$= \int_0^{\sqrt{\pi}} \frac{x}{3} \cos(x^2)\, dx$$

$$\overset{\substack{u=x^2 \\ du=2x\,dx \\ \frac{du}{2}=x\,dx}}{=} \int_0^{\pi} \frac{1}{6} \cos(u)\, du$$

$$= \frac{1}{6} \sin u \Big|_0^{\pi}$$

$$= \boxed{0}$$

Bonus. (5 pts) Find the volume of the unit ball $x^2 + y^2 + z^2 \leq 1$. (*Hint: view the upper hemisphere as a graph*)



Volume (ball)

$= 2 \cdot$ Volume (upper hemisphere)

$$= 2 \iint_D \sqrt{1-x^2-y^2}\, dA$$

$$= 2 \int_0^{2\pi} \int_0^1 \sqrt{1-r^2}\, r\, dr\, d\theta$$

$u = 1-r^2$

$du = -2r\,dr$

$\frac{du}{-2} = r\,dr$

$$= 2 \int_0^{2\pi} \left(\int_1^0 \sqrt{u}\, \frac{du}{-2} \right) d\theta$$

$$= \int_0^{2\pi} \left(\int_0^1 \sqrt{u}\, du \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{u^{3/2}}{3/2} \Big|_0^1 \right) d\theta$$

$$= \int_0^{2\pi} \frac{2}{3}\, d\theta$$

$$= \boxed{\frac{4\pi}{3}}$$