Math 234 Review

Chapter 2: Parametric curves and vector functions

1. Consider the helix given by

$$\vec{\mathbf{x}}(t) = \begin{pmatrix} R\cos(\omega t) \\ R\sin(\omega t) \\ at \end{pmatrix}.$$

(1) Compute $\vec{\mathbf{x}}'(t)$, $\vec{\mathbf{x}}''(t)$ and $\vec{\mathbf{x}}'''(t)$.
(2) Compute $\|\vec{\mathbf{x}}'(t)\|$.
(3) Find the length of the segment with $0 \le t \le \pi$.
(4) Find the angle between $\vec{\mathbf{x}}'(t)$ and $\vec{\mathbf{x}}''(t)$.
(5) Compute the triple product $\vec{\mathbf{x}}'(t) \cdot (\vec{\mathbf{x}}''(t) \times \vec{\mathbf{x}}'''(t))$.
(6) Find the unit tangent vector $\vec{\mathbf{T}}(t)$.
(7) Find the curvature vector $\vec{\kappa}(t)$ and curvature $\kappa(t)$.
(8) Find the normal vector $\vec{\mathbf{N}}(t)$.
(9) Find the binormal vector $\vec{\mathbf{B}}(t)$.
(10) Find the equation of the tangent line to the curve at time t.
(11) Find the intersection of the tangent line with the xy-plane.

2. Consider the curve given by

$$\vec{\mathbf{x}}(t) = \begin{pmatrix} t \\ t^3 \end{pmatrix}.$$

(1) Find the curvature of the curve at $t = 0$.
(2) Find the limit of the curvature as $t \to \infty$.

Chapter 3: Functions of more than one variable

1. Sketch the graph and level curves of the following functions.
(1) $f(x, y) = x^2 + y^2$
(2) $f(x, y) = xy$
(3) $f(x, y) = x^2$

2. (a) Find the equation of the plane that contains $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 1, 1)$. (b) Find the intersection of the plane with the y-axis. (c) Find a normal vector to the plane.

3. (a) Find the equation of the plane that contains the point $(1, 1, 1)$ and is normal to the vector $(1, 1, 1)$. (b) Find the intersection of the plane with the xy-plane.

4. (a) Classify the following quadratic forms as definite, indefinite, or semidefinite. (b) Determine their zero sets.
(1) $2x^2 + 4xy + 4y^2$

(2) $3x^2 + 6xy$
(3) $4x^2 - 8xy + 4y^2$

5. (a) Write the function $f(x, y) = \frac{y}{\sqrt{x^2+y^2}}$ in terms of polar coordinates. (b) Write the function $f(r, \theta) = r \tan \theta$ in terms of xy-coordinates. (c) Sketch the graph of the function $f(x, y) = \frac{1}{x^2+y^2}$.

Chapter 4: Derivatives

1. Find the partial derivatives f_x and f_y of the following functions.
(1) $f(x, y) = (x - y)^3$
(2) $f(x, y) = e^{-x^2-y^2}$
(3) $f(x, y) = \ln \sqrt{x^2 + y^2}$
(4) $f(x, y) = \frac{x}{x^2+y^2}$
(5) $f(x, y) = e^{xy} \cos(xy)$
(6) $f(x, y) = \arctan(\frac{y}{x})$

2. Let $f(x, y) = x^2y + xy^2$.
(1) Find an equation for the plane tangent to the graph of $f(x, y)$ at $(1, 2, 6)$.
(2) Find the linear approximation to $f(x, y)$ at the point $(1, 2)$.
(3) Find a normal vector to the level curve of $f(x, y)$ at $(1, 2)$.
(4) Find an equation for the tangent line to the level curve of $f(x, y)$ at $(1, 2)$.
(5) Find a direction in which $f(x, y)$ does not change at $(1, 2)$.

3. (a) Find an equation for the plane tangent to the surface defined by $x^2 - 2y^2 + 3z^2 = 2$ at $(1, -1, 1)$. (b) Find an equation for the normal line to the surface at $(1, -1, 1)$.

4. Suppose $y = f(x)$ satisfies the equation $x^2 - y^2 + y^3 = 0$. Use implicit differentiation to find the derivative of $f(x)$ at $(0, 1)$ and $(\sqrt{2}, -1)$ respectively.

5. Suppose $z = f(x, y)$ satisfies the equation $x^2 + y^2 - z^2 + z^3 = 0$. Use implicit differentiation to find the partial derivatives of $f(x, y)$ at $(1, 1, -1)$.

6. Find the second derivatives of the following functions.
(1) $f(x, y) = e^x \cos y$
(2) $f(x, y) = \ln(x^2 + y^2)$
(3) $f(x, y) = \arctan(\frac{y}{x})$
(4) $f(x, y) = \frac{y}{x^2+y^2}$
(5) $f(x, y, z) = x^2 + y^2 + z^2$

7. Find a function $f(x, y)$ (when exists) that satisfies the given equations.
(1) $f_x = x^2y, f_y = xy^2$
(2) $f_x = xy^2, f_y = x^2y$
(3) $f_x = ye^{xy}, f_y = xe^{xy}$