Chapter 5: Maxima and Minima

1. Find all critical points of the following functions. Apply the second derivative test to the critical points you find.

(a) $2x^{2} + 6xy + 5y^{2} + 6x + 10y + 5$ (b) $x^{2} + 3xy + 2y^{2} - x - 2$ (c) $3x^{2} + 2x^{3} + 2y^{2} + y^{4}$ (d) xy(3x + 3y + 1)(e) $x^{4} + y^{4} - 4xy$

2. Find all critical points of the function

$$f(x, y, z) = x^{2} + (x - y)^{2} + (y - z)^{2} + (z - 1)^{2}.$$

3. Find the volume of the largest rectangular box with edges parallel to the axes that can be inscribed in the surface

$$x^4 + y^4 + z^4 = 1.$$

4. Find all points on the surface

$$x^2 + 4y^2 + 9z^2 = 36$$

that are closest to the origin.

5.* A thin cylindrical drinking cup has a disk-shaped base and is of constant density 1 per unit area. The cup has total mass fixed to be S. Find the largest possible capacity of the cup.

Chapter 6: Integrals

- 1. Compute the following double integrals.
 - (a) $\iint_{D} \sin(x^{2}) dA, \text{ where } D = \{(x, y) : 0 \le y \le 1, y \le x \le 1\}$ (b) $\iint_{D} \frac{y}{1+x^{2}} dA, \text{ where } D = \{(x, y) : 0 \le y \le 1, y^{2} \le x \le 1\}$ (c) $\iint_{D} xe^{y} dA, \text{ where } D = \{(x, y) : 0 \le y \le 1, \sqrt{y} \le x \le y\}$ (d) $\iint_{D} \sqrt{4-x^{2}-y^{2}} dA, \text{ where } D = \{(x, y) : x^{2}+y^{2} \le 2\}$
- 2. Find the volume of the following regions by computing a double integral.
 - (a) the region in the first octant bounded by $y^2 = 4 x$, $y^2 = x$ and z = y.
 - (b) the region in the first octant bounded by $z^2 = x^2 + y^2$ and $z = x^2 + y^2$.
- 3. Check out the last homework assignment for triple integrals.