Math 234 Review

Chapter 6: Moment of Inertia, Cylindrical and Spherical Coordinates

1. Compute the moment of inertia of the wooden cylinder

$$D = \{(x, y, z) : x^2 + y^2 \leq 10000, 0 \leq z \leq 100\}$$

around the z-axis. The density of the wood is

$$\mu(x, y, z) = \frac{1}{x^2 + y^2} e^{-x^2-y^2-z}.$$

Hint: μ should be part of the integrand, since it is not constant.

2. Compute the moment of inertia of the ice cream

$$D = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1, z \geq \sqrt{x^2 + y^2}\}$$

around the z-axis. The density of the ice cream is $\mu = 1$.
Hint: $\sin^3 \phi = \sin^2 \phi \sin \phi = (1 - \cos^2 \phi) \sin \phi$.

Chapter 7: Vector Calculus

1. If $\mathcal{C}$ is the helix given by

$$\overrightarrow{\mathbf{x}}(t) = (\cos t, \sin t, t),\ \ 0 \leq t \leq 1000.$$

What is the average of $f = \cos \theta$ on $\mathcal{C}$? Where θ is the angle between the position vector and the z-axis.
Hint: $\cos \theta = \frac{\overrightarrow{\mathbf{x}} \cdot \overrightarrow{\mathbf{e}_3}}{\|\overrightarrow{\mathbf{x}}\| \|\overrightarrow{\mathbf{e}_3}\|}$.

2. Let $\mathcal{C}$ be the boundary of the ice cream

$$D = \{(x, y) : x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}.$$

Find the average x and y coordinates on $\mathcal{C}$.

3. If $\mathcal{C}$ is the curve given by

$$\overrightarrow{\mathbf{x}}(t) = (t, t^2, t^3),\ \ 0 \leq t \leq 1.$$

$\overrightarrow{\mathbf{F}}$ is the vector field $x\overrightarrow{\mathbf{e}}_1 + y\overrightarrow{\mathbf{e}}_2 + z\overrightarrow{\mathbf{e}}_3$. Compute the line integral

$$\int_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{x}}.$$

4. If $\mathcal{C}$ is the counter-clockwise traversed boundary of the region

$$D = \{(x, y) : 0 \leq x, y \leq 1\}.$$

$\overrightarrow{\mathbf{F}}(x,y) = x^2y\,\overrightarrow{\mathbf{e}}_1 + xy^2\,\overrightarrow{\mathbf{e}}_2$. (1) Compute the line integral

$$\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot d\overrightarrow{\mathbf{x}}$$

in two ways: directly, and by using Green's Theorem. (2) Is $\overrightarrow{\mathbf{F}}$ a conservative vector field? (3) Compute the flux integral

$$\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} ds$$

where $\overrightarrow{\mathbf{N}}$ is the outward normal.

5. If $\mathcal{C}$ is the clockwise traversed boundary of the arch

$$D = \{(x,y) : 1 \le x^2 + y^2 \le 4, y \ge 0\}.$$

$\overrightarrow{\mathbf{F}}(x,y) = (-xy^2, x^2y)$. (1) Compute the line integrals in two ways

$$\oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds \quad \text{and} \quad \oint_{\mathcal{C}} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} ds.$$

(2) Is $\overrightarrow{\mathbf{F}}$ conservative or divergence-free?

6. Compute

$$\oint_{\mathcal{C}} (y^2 + x)e^x dx$$

where $\mathcal{C}$ is the clockwise traversed boundary of the region

$$D = \{(x,y) : 0 \le x \le 1, \sqrt{x} \le y \le 1\}.$$

7.* Check out the last homework assignment for surface integrals.