## Math 234 Discussion Worksheet - Sep 25

**1.** Let $f(x,y) = \arctan(\frac{y}{x})$.

(1) Compute the partial derivatives $f_x$ and $f_y$.
(2) Find the linear approximation of $f$ near $(1,1)$.
(3) Find the tangent plane to the graph of $f$ at $(1,1)$.
(4) Find the gradient of $f$ at $(1,1)$.
(5) What is the level curve of $f$ that passes through $(1,1)$?
(6) How is the gradient you found in (4) related to the level curve in (5)?

(1) $\frac{\partial}{\partial x}\left(\arctan\left(\frac{y}{x}\right)\right) = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{-y}{x^2} = \frac{-y}{\left(1+\frac{y^2}{x^2}\right)x^2} = \boxed{\frac{-y}{x^2+y^2}}$

$\frac{\partial}{\partial y}\left(\arctan\left(\frac{y}{x}\right)\right) = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} = \frac{x}{\left(1+\frac{y^2}{x^2}\right)x^2} = \boxed{\frac{x}{x^2+y^2}}$

(2) $f(x,y) \approx f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \boxed{\frac{\pi}{4} + \left(-\frac{1}{2}\right)(x-1) + \frac{1}{2}(y-1)}$

(3) $z = \boxed{\frac{\pi}{4} - \frac{1}{2}x + \frac{1}{2}y}$ (from (2))

(4) $\nabla f(1,1) = (f_x(1,1), f_y(1,1))$

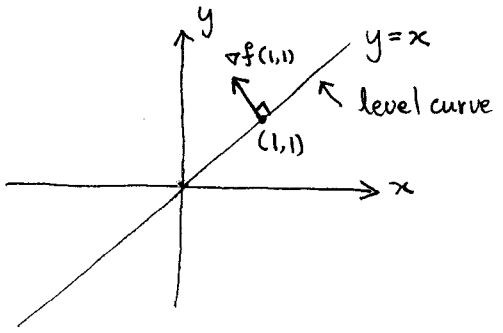
$= \boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$ (from (1))

(5) at $(1,1)$, $f(1,1) = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$

So the level curve is given by $\arctan\left(\frac{y}{x}\right) = \frac{\pi}{4}$,

i.e. $\frac{y}{x} = 1$, or $\boxed{y=x}$ (a line with slope 1)

(6)




(1) $f_x = -\frac{y}{x^2+y^2}$, $f_y = \frac{x}{x^2+y^2}$
(2) $f(x,y) \approx \frac{\pi}{4} - \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$
(3) $z = \frac{\pi}{4} - \frac{1}{2}(x-1) + \frac{1}{2}(y-1)$
(4) $\nabla f(1,1) = (-\frac{1}{2}, \frac{1}{2})$
(5) the line $y = x$
(6) perpendicular