Math 234 Discussion Worksheet - Oct 2

1. Find an equation for the plane tangent to the surface $x^2 + y^2 + z^2 + z^3 = 2$ at the point $(1, 1, -1)$.

2. Suppose $y = y(x)$ is implicitly defined by the equation $y + e^y = x$ near the point $(1, 0)$. Compute $y'(1)$.

3. Suppose $z = z(x, y)$ is implicitly defined by the equation $x^2 + y^2 + z^2 + z^3 = 2$ near the point $(1, 1, -1)$. (a) Compute $z_x(1, 1)$ and $z_y(1, 1)$. (b) Find an equation for the plane tangent to the graph of $z(x, y)$ at the point $(1, 1, -1)$.

4. Compute the second derivatives of the function $f(x, y) = \sin(xy)$.

1. Let $f(x,y,z) = x^2+y^2+z^2+z^3$, then

$\vec{\nabla} f(x,y,z) = (2x, 2y, 2z+3z^2)$

$\vec{\nabla} f(1,1,-1) = (2,2,1)$ (normal to the surface at $(1,1,-1)$)

$\vec{\nabla} f(1,1,-1) \cdot (\vec{x} - \vec{x_0}) = 0$ (the eqn. of the tangent plane at $(1,1,-1) = \vec{x_0}$)

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right] = 0$$

$$\boxed{2(x-1) + 2(y-1) + 1(z+1) = 0}$$

2. $\frac{d}{dx}\left(y(x) + e^{y(x)} = x \right)$

$\rightsquigarrow y'(x) + e^{y(x)} y'(x) = 1$

at $(x,y) = (1,0)$, this becomes

$y'(1) + e^0 y'(1) = 1$

$\Rightarrow y'(1) = \boxed{\frac{1}{2}}$

3.(a) $\frac{\partial}{\partial x}\left(x^2+y^2+z(x,y)^2 + z(x,y)^3 = 2 \right)$

$\rightsquigarrow 2x + 2z(x,y)\, z_x(x,y) + 3z(x,y)^2 z_x(x,y) = 0$

at $(x,y,z) = (1,1,-1)$, this becomes

$2 - 2z_x(1,1) + 3z_x(1,1) = 0$

$\Rightarrow z_x(1,1) = \boxed{-2}$

Similarly $z_y(1,1) = \boxed{-2}$

4. $f_x(x,y) = y\cos(xy)$

$f_y(x,y) = x\cos(xy)$

$f_{xx}(x,y) = \frac{\partial}{\partial x}\left(y\cos(xy) \right)$

$= \boxed{-y^2 \sin(xy)}$

$f_{xy}(x,y) = \frac{\partial}{\partial x}\left(x\cos(xy) \right)$

$= \cos(xy) + (-y)x\sin(xy)$

$= \boxed{\cos(xy) - xy\sin(xy)}$

$f_{yy}(x,y) = \boxed{-x^2 \sin(xy)}$

(1) $2(x-1) + 2(y-1) + (z+1) = 0$

(2) $\frac{1}{2}$

(3) (a) $-2, -2$

(b) $z = -1 - 2(x-1) - 2(y-1)$

(3) $y^2\sin(xy), \cos(xy) - xy\sin(xy), x^2\sin(xy)$

same

3.(b) $z = z(1,1) + z_x(1,1)(x-1) + z_y(1,1)(y-1)$

$= \boxed{-1 - 2(x-y) - 2(y-1)}$