Math 234 Discussion Worksheet - Oct 20

1. Find all critical points of the following functions, and apply the second derivative test to the points you find.

(a) $f(x,y) = x^2 + 2y^2 - x^2y$.

(b) $f(x,y) = 2x + y - x^2y$.

2. Find the minimum of the function $f(x,y) = (1 - x^2 - y^2)^2$.

1. (a) $\begin{cases} f_x = 2x - 2xy = 0 \\ f_y = 4y - x^2 = 0 \end{cases}$ or $\begin{cases} 2x(1-y) = 0 \cdots (1) \\ 4y - x^2 = 0 \cdots (2) \end{cases}$

From (1), we get $x=0$ or $y=1$.

If $x=0$, from (2) we get $y=0$

If $y=1$, from (2) we get $4=x^2$, i.e. $x=\pm 2$.

So the critical points are: $\boxed{(0,0),\ (2,1),\ (-2,1)}$.

$f_{xx} = 2-2y \quad f_{xy} = -2x \quad f_{yy} = 4$

@ $(0,0)$, get $\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$, So it's a $\boxed{\text{loc min.}}$

@ $(2,1)$, get $\begin{pmatrix} 0 & -4 \\ -4 & 4 \end{pmatrix}$, So it's a $\boxed{\text{Saddle point.}}$

@ $(-2,1)$, get $\begin{pmatrix} 0 & 4 \\ 4 & 4 \end{pmatrix}$, So it's a $\boxed{\text{Saddle point.}}$

(b) $\begin{cases} f_x = 2 - 2xy = 0 \cdots (1) \\ f_y = 1 - x^2 = 0 \cdots (2) \end{cases}$

From (2), we get $x = \pm 1$.

If $x=1$, we get $y=1$

If $x=-1$, we get $y=-1$.

So the critical points are $\boxed{(1,1) \text{ and } (-1,-1)}$

@ $(1,1)$, we get $\begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix} = \begin{pmatrix} -2y & -2x \\ -2x & 0 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -2 & 0 \end{pmatrix}$, So it's a $\boxed{\text{Saddle}}$

@ $(-1,-1)$, we get $\begin{pmatrix} 2 & 2 \\ 2 & 0 \end{pmatrix}$, So it's a $\boxed{\text{Saddle}}$.

2. $f(x,y) = u^2$ where $u = (1-x^2-y^2)$. So f is minimized when $u=0$,

i.e. $\boxed{1-x^2-y^2=0}$ (unit circle), and the minimum is $\boxed{0}$.