

## Math 234 Discussion Worksheet - Nov 4

1. Compute the following double integrals.

(a)  $\iint_D \frac{\sin x}{x} dA$ , where  $D = \{(x, y) : 0 \leq y \leq \pi, y \leq x \leq \pi\}$

(b)  $\iint_D 4ye^{-x^2} dA$ , where  $D = \{(x, y) : 0 \leq y \leq 1, y^2 \leq x \leq 1\}$

(c)  $\iint_D e^{-x^2-y^2} dA$ , where  $D = \{(x, y) : x^2 + y^2 \leq R\}$

2. Find the volume of the region in the first octant bounded by  $z = 1 - x^2 - y^2$  and  $x + y = 1$ .

$$1. (a) \iint_D \frac{\sin x}{x} dA = \int_0^\pi \int_0^x \frac{\sin x}{x} dy dx = \int_0^\pi \sin x dx = \boxed{2}$$

$$1. (b) \iint_D 4ye^{-x^2} dA = \int_0^1 \int_0^{\sqrt{x}} 4ye^{-x^2} dy dx = \int_0^1 2xe^{-x^2} dx = \boxed{1 - e^{-1}}$$

$$1. (c) \iint_D e^{-x^2-y^2} dA = \int_0^R \int_0^{2\pi} e^{-r^2} r d\theta dr = 2\pi \int_0^R e^{-r^2} r dr = \boxed{\pi(1 - e^{-R})}$$

2. The volume is given by the double integral

$$\iint_D 1 - x^2 - y^2 dA, \text{ where } D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1 - x\}$$

which can be written as the iterated integral

$$\begin{aligned} & \int_0^1 \int_0^{1-x} (1 - x^2 - y^2) dy dx \\ &= \int_0^1 \left( y - x^2 y - \frac{y^3}{3} \right) \Big|_0^{1-x} dx \\ &= \int_0^1 \left\{ (1-x) - x^2(1-x) - \frac{(1-x)^3}{3} \right\} dx \\ &= \int_0^1 \left\{ u - (1-u)^2 u - \frac{u^3}{3} \right\} du \quad (u = 1-x) \\ &= \int_0^1 \left\{ u - (u - 2u^2 + u^3) - \frac{u^3}{3} \right\} du \\ &= \int_0^1 \left\{ 2u^2 - \frac{4u^3}{3} \right\} du \\ &= \frac{2}{3} - \frac{1}{3} \\ &= \boxed{\frac{1}{3}} \end{aligned}$$