Math 234 Discussion Worksheet - Nov 20

1. Compute $\int_C y ds$ where C is the curve $y = x^3, 0 \leq x \leq 1$.
2. Suppose a wire C is the quarter of the unit circle in the first quadrant, and is of constant density 1. Find the center of mass of the wire.
3. Compute $\oint_C x dx + y dy$ and $\oint_C -y dx + x dy$ where C is the unit circle.

1. See quiz 9

2. See quiz 9

3. C can be parametrized by $\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$ ($x(t)$, $y(t)$), $0 \leq t \leq 2\pi$.

So $$\oint_C x dx + y dy = \int_0^{2\pi} x(t)\, dx(t) + y(t)\, dy(t)$$

$$= \int_0^{2\pi} \cos t (-\sin t)\, dt + \sin t (\cos t)\, dt$$

$$= \int_0^{2\pi} (-\cos t \sin t + \sin t \cos t)\, dt$$

$$= \int_0^{2\pi} 0\, dt = \boxed{0}$$

$$\oint_C -y dx + x dy = \int_0^{2\pi} -\sin t\, d(\cos t) + \cos t\, d(\sin t)$$

$$= \int_0^{2\pi} -\sin t (\sin t)\, dt + \cos t (\cos t)\, dt$$

$$= \int_0^{2\pi} \left[(\sin t)^2 + (\cos t)^2 \right] dt$$

$$= \int_0^{2\pi} 1\, dt$$

$$= \boxed{2\pi}$$