Math 276 Discussion Worksheet 1

1. Compute $f'(x)$ where f is defined by the formula

$$f(x) = \int_{-x^2}^{x^2} t^3 dt.$$

2. Suppose f is continuous everywhere. Show that

$$F(x) = \int_0^x (x - t) f(t) dt.$$

is twice differentiable and

$$F''(x) = f(x).$$

3. Suppose f is $(n + 1)$ times differentiable. Prove that f is a polynomial of the form $f(x) = a_0 + a_1 x + \cdots + a_n x^n$ if and only if its $(n + 1)$-th derivative $f^{(n+1)}(x) = 0$.

Hints

1. Write $\int_{-x^2}^{x^2} = \int_{-x^2}^{0} + \int_{0}^{x^2}$, and then apply the chain rule with $u = x^2$.

2. Write $(x - t)f(t) = xf(t) - tf(t)$.

3. Start with the case $n = 0$ and then proceed by induction.