Math 276 Discussion Worksheet 17

1. Find $f'(x)$ if

$$f(x) = \int_{\arctan(x^2)}^{\arccos(x^2)} u\, du$$

2. Compute the limits

$$\lim_{x \to 0} \frac{\cos(x) - 1 + x^2/2}{\tan^4 x}, \quad \lim_{x \to \infty} \left((1 - e^{-x})^{-e^x} + \frac{\cos(e^x)x^{100}}{2^x} \right)$$

3. Find the following antiderivative (assuming $x > 1$)

$$\int \frac{1}{x(\log x)} dx$$

4. Compute the integral

$$\int_0^{\pi} x^2 (\cos x) dx$$

5. Study convergence of the series

$$\sum_{n=2}^{\infty} \left(\frac{(-1)^n}{n(\log n)^\alpha} + \frac{1}{2^n} \right)$$

for all values of $\alpha \in \mathbb{R}$. If the series converges for some α, does it converge absolutely?

6*. Let

$$S(x) = \sum_{n=1}^{\infty} \frac{\sin^2(nx)}{n^2}$$

Find

$$\int_0^{\pi} S(x) dx$$

and justify your answers. (Hint: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$)

7. Find the radius of convergence for the following power series

$$\sum_{n=1}^{\infty} \left((-1)^n + \log n \right) x^{5n}$$

Study the convergence of this series on the boundary of the interval of convergence.

8. Find the power series expansion for $f(x)$ if

$$f(x) = \cos(x^2) + \frac{x}{1 - x}$$

What is the radius of convergence.

9. 10. 11.