231 Gateway 3 Practice Test - Graphs

No uses of Calculators; No Partial Credit. 30 minutes to finish test. More space will be provided on the actual test.

1. (20 pts) Find the critical numbers and give the values of the absolute minimum and maximum for the function $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x$ on the interval [-1, 2].

2. (10 pts) Find points where f has a local maximum or minimum on the given domain and identify each point as a local maximum or local minimum. If there is no local maximum or minimum, explain (briefly) why.

$$f(x) = x^2 + \frac{3}{x}, 0 < x < \infty.$$

3. (28 pts) For the given <u>derivative</u> of a function f, f'(x) = (x+1)(x+2),

(a) What are the critical numbers of f?

- (b) On what intervals is f increasing?
- (c) On what intervals is f decreasing?

(d) At what points, if any, does f assume a local maximum or local minimum values?

4. (12 pts) The graphs of the first and second derivative of a function y = f(x) are shown. Add to the picture a sketch of the approximate graph of f, given that the graph passes through the point P.



5. (20 pts) The accompanying figure shows a portion of the graph of a twice-differentiable function y = f(x). At each of the five labeled points, classify y' and y'' as positive, negative, or zero.



6. Use the following information to answer this problem:

$$\begin{split} f(-1) & \text{does not exist} \\ \lim_{x \to -\infty} f(x) &= -1 \\ \lim_{x \to \infty} f(x) &= 1 \\ f'(x) &< 0 \text{ for } x \in (-\infty, -1) \\ f'(x) &> 0 \text{ for } x \in (-1, \infty) \\ f''(x) &< 0 \text{ for } x \in (-\infty, -2) \cup (0, \infty) \\ f''(x) &> 0 \text{ for } x \in (-2, -1) \cup (-1, 0) \end{split}$$

a) (4 pts) Label all of the important x-values derived from the information provided above on the number line below. Then indicate the intervals along this line where f'(x) and f''(x) are positive or negative.

 $f'(x) \xrightarrow{} f''(x)$

b) (6 pts) Sketch the graph of a function that satisfies all of the given conditions. No formulas are required but be sure to label all of the important values on the coordinate axes.