## 231 Gateway 3 Practice Test - Graphs

No uses of Calculators; No Partial Credit. 30 minutes to finish test. More space will be provided on the actual test.

1. $(20 \mathrm{pts})$ Find the critical numbers and give the values of the absolute minimum and maximum for the function $f(x)=3 x^{4}+8 x^{3}-6 x^{2}-24 x$ on the interval $[-1,2]$.
2. ( 10 pts ) Find points where $f$ has a local maximum or minimum on the given domain and identify each point as a local maximum or local minimum. If there is no local maximum or minimum, explain (briefly) why.
$f(x)=x^{2}+\frac{3}{x}, 0<x<\infty$.
3. $(28 \mathrm{pts})$ For the given derivative of a function $f, f^{\prime}(x)=(x+1)(x+2)$,
(a) What are the critical numbers of $f$ ?
(b) On what intervals is $f$ increasing?
(c) On what intervals is $f$ decreasing?
(d) At what points, if any, does $f$ assume a local maximum or local minimum values?
4. (12 pts) The graphs of the first and second derivative of a function $y=f(x)$ are shown. Add to the picture a sketch of the approximate graph of $f$, given that the graph passes through the point $P$.

5. (20 pts) The accompanying figure shows a portion of the graph of a twice-differentiable function $y=f(x)$. At each of the five labeled points, classify $y^{\prime}$ and $y^{\prime \prime}$ as positive, negative, or zero.


| Point | $y^{\prime}$ | $y^{\prime \prime}$ |
| :---: | :---: | :---: |
| P |  |  |
| Q |  |  |
| R |  |  |
| S |  |  |
| T |  |  |

6. Use the following information to answer this problem:
$f(-1)$ does not exist
$\lim _{x \rightarrow-\infty} f(x)=-1$
$\lim _{x \rightarrow \infty} f(x)=1$
$f^{\prime}(x)<0$ for $x \in(-\infty,-1)$
$f^{\prime}(x)>0$ for $x \in(-1, \infty)$
$f^{\prime \prime}(x)<0$ for $x \in(-\infty,-2) \cup(0, \infty)$
$f^{\prime \prime}(x)>0$ for $x \in(-2,-1) \cup(-1,0)$
a) (4 pts) Label all of the important $x$-values derived from the information provided above on the number line below. Then indicate the intervals along this line where $f^{\prime}(x)$ and $f^{\prime \prime}(x)$ are positive or negative.

$$
\begin{aligned}
& f^{\prime}(x) \\
& f^{\prime \prime}(x)
\end{aligned}
$$

b) ( 6 pts) Sketch the graph of a function that satisfies all of the given conditions. No formulas are required but be sure to label all of the important values on the coordinate axes.

