1 (1) F (2) F (3) T (4) T (5) F (6) F (7) T (8) T (9) F (10) F
(11) T (12) **F** (13) **F** (14) T (15) F (16) T (17) F (18) **F**

2(1) $\lim\limits_{x\to -3} \dfrac{x^2-9}{x^2+2x-3} = \lim\limits_{x\to -3} \dfrac{(x+3)(x-3)}{(x+3)(x-1)} = \lim\limits_{x\to -3} \dfrac{x-3}{x-1} = \dfrac{-3-3}{-3-1} = \dfrac{-6}{-4} = \dfrac{3}{2}$

2(2) $\lim\limits_{x\to -\infty} \dfrac{1-2x^2-x^4}{5+x-3x^4} = \lim\limits_{x\to -\infty} \dfrac{(1-2x^2-x^4)/x^4}{(5+x-3x^4)/x^4} = \lim\limits_{x\to -\infty} \dfrac{1/x^4-2/x^2-1}{5/x^4+1/x^3-3} = \dfrac{0-0-1}{0+0-3} = \dfrac{-1}{-3} = \dfrac{1}{3}$

2(3) $\lim\limits_{x\to\infty} \left(\sqrt{x^2+4x+1}-x\right) = \lim\limits_{x\to\infty} \left[\dfrac{\sqrt{x^2+4x+1}-x}{1} \cdot \dfrac{\sqrt{x^2+4x+1}+x}{\sqrt{x^2+4x+1}+x}\right] = \lim\limits_{x\to\infty} \dfrac{(x^2+4x+1)-x^2}{\sqrt{x^2+4x+1}+x}$

$= \lim\limits_{x\to\infty} \dfrac{(4x+1)/x}{(\sqrt{x^2+4x+1}+x)/x}$ $\quad \left[\text{divide by } x=\sqrt{x^2} \text{ for } x>0\right]$

$= \lim\limits_{x\to\infty} \dfrac{4+1/x}{\sqrt{1+4/x+1/x^2}+1} = \dfrac{4+0}{\sqrt{1+0+0}+1} = \dfrac{4}{2} = 2$

2(4) $= \lim\limits_{s\to 16} \dfrac{4-\sqrt{s}}{(\sqrt{s}+4)(\sqrt{s}-4)} = \lim\limits_{s\to 16} \dfrac{-1}{\sqrt{s}+4} = \dfrac{-1}{\sqrt{16}+4} = -\dfrac{1}{8}$

2(5) $\lim\limits_{x\to 0} \cos(x+\sin x) = \cos\left[\lim\limits_{x\to 0}(x+\sin x)\right]$ [by continuity] $= \cos 0 = 1$

3(1) $y = \dfrac{x^2-x+2}{\sqrt{x}} = x^{3/2}-x^{1/2}+2x^{-1/2} \quad\Rightarrow\quad y' = \dfrac{3}{2}x^{1/2}-\dfrac{1}{2}x^{-1/2}-x^{-3/2} = \dfrac{3}{2}\sqrt{x}-\dfrac{1}{2\sqrt{x}}-\dfrac{1}{\sqrt{x^3}}$

3(2) $y = x^2\sin\pi x \quad\Rightarrow\quad y' = x^2(\cos\pi x)\pi + (\sin\pi x)(2x) = x(\pi x\cos\pi x + 2\sin\pi x)$

3(3) $y = \dfrac{\tan x}{1+\cos x} \quad\Rightarrow\quad y' = \dfrac{(1+\cos x)\sec^2 x - \tan x(-\sin x)}{(1+\cos x)^2} = \dfrac{(1+\cos x)\sec^2 x + \tan x\,\sin x}{(1+\cos x)^2}$

3(4) $y = \sin(\cos x) \quad\Rightarrow\quad y' = \cos(\cos x)(-\sin x) = -\sin x\,\cos(\cos x)$

3(5) $y = \ln(x\ln x) \quad\Rightarrow\quad y' = \dfrac{1}{x\ln x}(x\ln x)' = \dfrac{1}{x\ln x}\left(x\cdot\dfrac{1}{x}+\ln x\cdot 1\right) = \dfrac{1+\ln x}{x\ln x}$

Another method: $y = \ln(x\ln x) = \ln x + \ln\ln x \quad\Rightarrow\quad y' = \dfrac{1}{x}+\dfrac{1}{\ln x}\cdot\dfrac{1}{x} = \dfrac{\ln x+1}{x\ln x}$

3(6) $h(\theta) = e^{\tan 2\theta} \quad\Rightarrow\quad h'(\theta) = e^{\tan 2\theta}\cdot\sec^2 2\theta\cdot 2 = 2\sec^2(2\theta)\,e^{\tan 2\theta}$

3(7) $g(t) = \dfrac{e^t}{1+e^t} \quad\Rightarrow\quad g'(t) = \dfrac{(1+e^t)e^t - e^t(e^t)}{(1+e^t)^2} = \dfrac{e^t}{(1+e^t)^2}$

3(8) $y = (\cos x)^x \quad\Rightarrow\quad \ln y = \ln(\cos x)^x = x\ln\cos x \quad\Rightarrow\quad \dfrac{y'}{y} = x\cdot\dfrac{1}{\cos x}\cdot(-\sin x)+\ln\cos x\cdot 1 \quad\Rightarrow$

$y' = (\cos x)^x(\ln\cos x - x\tan x)$

3(9) $y = 3^{x\ln x} \quad\Rightarrow\quad y' = 3^{x\ln x}(\ln 3)\dfrac{d}{dx}(x\ln x) = 3^{x\ln x}(\ln 3)\quad x\cdot\dfrac{1}{x}+\ln x\cdot 1\Big) = 3^{x\ln x}(\ln 3)(1+\ln x)$

3(10) $y = \sqrt{\arctan x} \quad \Rightarrow \quad y' = \frac{1}{2}(\arctan x)^{-1/2}\,\frac{d}{dx}\,(\arctan x) = \frac{1}{2\sqrt{\arctan x}\,(1+x^2)}$

3(11) $y = (\arcsin 2x)^2 \quad \Rightarrow \quad y' = 2(\arcsin 2x)\cdot(\arcsin 2x)' = 2\arcsin 2x \cdot \frac{1}{\sqrt{1-(2x)^2}}\cdot 2 = \frac{4\arcsin 2x}{\sqrt{1-4x^2}}$

3(12) $y = \sin^{-1}(e^x) \quad \Rightarrow \quad y' = 1/\sqrt{1-(e^x)^2}\cdot e^x = e^x/\sqrt{1-e^{2x}}$

4(1) $y = \sqrt{1+4\sin x} \quad \Rightarrow \quad y' = \frac{1}{2}(1+4\sin x)^{-1/2}\cdot 4\cos x = \frac{2\cos x}{\sqrt{1+4\sin x}}.$

At $(0,1)$, $y' = \frac{2}{\sqrt{1}} = 2$, so an equation of the tangent line is $y-1 = 2(x-0)$, or $y = 2x+1$.

The slope of the normal line is $-\frac{1}{2}$, so an equation of the normal line is $y-1 = -\frac{1}{2}(x-0)$, or $y = -\frac{1}{2}x+1$.

4(2) $x^2+4xy+y^2 = 13 \quad \Rightarrow \quad 2x+4(xy'+y\cdot 1)+2yy' = 0 \quad \Rightarrow \quad x+2xy'+2y+yy' = 0 \quad \Rightarrow$

$2xy'+yy' = -x-2y \quad \Rightarrow \quad y'(2x+y) = -x-2y \quad \Rightarrow \quad y' = \frac{-x-2y}{2x+y}.$

At $(2,1)$, $y' = \frac{-2-2}{4+1} = -\frac{4}{5}$, so an equation of the tangent line is $y-1 = -\frac{4}{5}(x-2)$, or $y = -\frac{4}{5}x+\frac{13}{5}$.

The slope of the normal line is $\frac{5}{4}$, so an equation of the normal line is $y-1 = \frac{5}{4}(x-2)$, or $y = \frac{5}{4}x-\frac{3}{2}$.

5 (1) $g(x) = 200+8x^3+x^4 \Rightarrow g'(x) = 24x^2+4x^3 = 4x^2(6+x) = 0$ when $x=-6$ and when $x=0$.

$g'(x) > 0 \quad \Leftrightarrow \quad x > -6 \quad [x \neq 0]$ and $g'(x) < 0 \quad \Leftrightarrow \quad x < -6$, so g is decreasing on $(-\infty,-6)$ and g is increasing on $(-6,\infty)$, with a horizontal tangent at $x=0$.

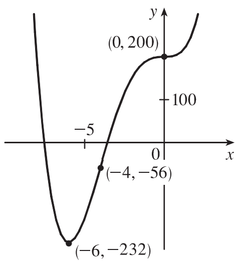
(2) $g(-6) = -232$ is a local minimum value.

There is no local maximum value.

(3) $g''(x) = 48x+12x^2 = 12x(4+x) = 0$ when $x=-4$ and when $x=0$.

$g''(x) > 0 \quad \Leftrightarrow \quad x < -4$ or $x > 0$ and $g''(x) < 0 \quad \Leftrightarrow \quad -4 < x < 0$, so g is CU on $(-\infty,-4)$ and $(0,\infty)$, and g is CD on $(-4,0)$. There are inflection points at $(-4,-56)$ and $(0,200)$.

(4)



6(1) $g(t) = \dfrac{1+t+t^2}{\sqrt{t}} = t^{-1/2} + t^{1/2} + t^{3/2} \quad \Rightarrow \quad G(t) = 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C$

6(2) $f(x) = \dfrac{x^5 - x^4 + 2x}{x^4} = x - 1 + \dfrac{2}{x^3} = x - 1 + 2x^{-3} \quad \Rightarrow \quad F(x) = \dfrac{x^2}{2} - x + 2\left(\dfrac{x^{-3+1}}{-3+1}\right) + C_1 = \frac{1}{2}x^2 - x - \dfrac{1}{x^2} + C_1$

on $(0, \infty)$ and $F(x) = \frac{1}{2}x^2 - x - \dfrac{1}{x^2} + C_2$ on $(-\infty, 0)$

6(3) Let $u = y^2 + 1$, so $du = 2y\,dy$ and $y\,dy = \frac{1}{2}\,du$. When $y = 0$, $u = 1$; when $y = 1$, $u = 2$. Thus,

$\int_0^1 y(y^2+1)^5\ dy = \int_1^2 u^5\left(\frac{1}{2}\,du\right) = \frac{1}{2}\left[\frac{1}{6}u^6\right]_1^2 = \frac{1}{12}(64-1) = \frac{63}{12} = \frac{21}{4}.$

6(4) Let $u = 1 + \tan t$, so $du = \sec^2 t\,dt$. When $t = 0$, $u = 1$; when $t = \frac{\pi}{4}$, $u = 2$. Thus,

$\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t\,dt = \int_1^2 u^3\,du = \left[\frac{1}{4}u^4\right]_1^2 = \frac{1}{4}\left(2^4 - 1^4\right) = \frac{1}{4}(16-1) = \frac{15}{4}.$

6(5) Let $u = \sin \pi t$. Then $du = \pi\ \cos \pi t\,dt$, so $\int \sin \pi t\ \cos \pi t\,dt = \int u\left(\frac{1}{\pi}\,du\right) = \frac{1}{\pi} \cdot \frac{1}{2}u^2 + C = \frac{1}{2\pi}(\sin \pi t)^2 + C.$

6(6) Let $u = x^2 + 2x$. Then $du = (2x+2)\,dx = 2(x+1)\,dx$ and

$\displaystyle\int \frac{x+1}{x^2+2x}\,dx = \int \frac{\frac{1}{2}\,du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left|x^2+2x\right| + C.$

6(7) Let $t = 4u$. Then $dt = 4\,du$ and

$$\int_0^4 \frac{1}{16+t^2}\,dt = \int_0^1 \frac{1}{16+16u^2} \cdot 4\,du = \frac{1}{4}\int_0^1 \frac{du}{1+u^2} = \frac{1}{4}\Big[\tan^{-1} u\Big]_0^1$$
$$= \tfrac{1}{4}(\tan^{-1} 1 - \tan^{-1} 0) = \tfrac{1}{4}\left(\tfrac{\pi}{4} - 0\right) = \tfrac{\pi}{16}$$

6(8) Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1} u + C = \frac{1}{2}\sin^{-1}\left(x^2\right) + C.$

6(9) $\displaystyle\int_0^2 \frac{dx}{e^{\pi x}} = \int_0^2 e^{-\pi x}\,dx = \left[-\frac{1}{\pi}e^{-\pi x}\right]_0^2 = -\frac{1}{\pi}e^{-2\pi} + \frac{1}{\pi}e^0 = \frac{1}{\pi}(1 - e^{-2\pi})$

6(10) Let $u = 5x + 1$, so $du = 5\,dx$. When $x = 0$, $u = 1$; when $x = 3$, $u = 16$. Thus,

$\displaystyle\int_0^3 \frac{dx}{5x+1} = \int_1^{16} \frac{1}{u}\left(\frac{1}{5}\,du\right) = \frac{1}{5}\Big[\ln|u|\Big]_1^{16} = \frac{1}{5}(\ln 16 - \ln 1) = \frac{1}{5}\ln 16.$

6(11) Let $u = 2 + \sin x$. Then $du = \cos x\,dx$, so

$\displaystyle\int \frac{\cos x}{2+\sin x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|2+\sin x| + C = \ln(2+\sin x) + C \quad [\text{since } 2 + \sin x > 0].$

7(1) $a(t) = v'(t) = 10\sin t + 3\cos t \quad \Rightarrow \quad v(t) = -10\cos t + 3\sin t + C \quad \Rightarrow \quad s(t) = -10\sin t - 3\cos t + Ct + D.$

$s(0) = -3 + D = 0$ and $s(2\pi) = -3 + 2\pi C + D = 12 \quad \Rightarrow \quad D = 3$ and $C = \frac{6}{\pi}$. Thus,

$s(t) = -10\sin t - 3\cos t + \frac{6}{\pi}t + 3.$

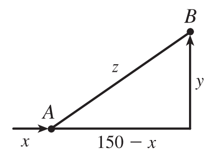
7(2) $a(t) = t^2 - 4t + 6 \quad \Rightarrow \quad v(t) = \frac{1}{3}t^3 - 2t^2 + 6t + C \quad \Rightarrow \quad s(t) = \frac{1}{12}t^4 - \frac{2}{3}t^3 + 3t^2 + Ct + D.$ $s(0) = D$ and

$s(0) = 0 \quad \Rightarrow \quad D = 0.$ $s(1) = \frac{29}{12} + C$ and $s(1) = 20 \quad \Rightarrow \quad C = \frac{211}{12}$. Thus, $s(t) = \frac{1}{12}t^4 - \frac{2}{3}t^3 + 3t^2 + \frac{211}{12}t.$

8 (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

(c)

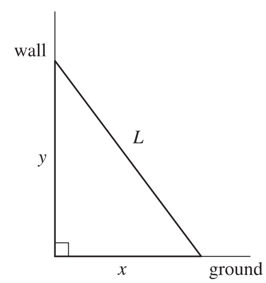


(d) $z^2 = (150 - x)^2 + y^2 \quad \Rightarrow \quad 2z\frac{dz}{dt} = 2(150 - x)\left(-\frac{dx}{dt}\right) + 2y\frac{dy}{dt}$

(e) At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \quad \Rightarrow \quad z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10{,}100}$.

So $\dfrac{dz}{dt} = \dfrac{1}{z}\left[(x - 150)\dfrac{dx}{dt} + y\dfrac{dy}{dt}\right] = \dfrac{-10(35) + 100(25)}{\sqrt{10{,}100}} = \dfrac{215}{\sqrt{101}} \approx 21.4$ km/h.

9 From the figure and given information, we have $x^2 + y^2 = L^2$, $\dfrac{dy}{dt} = -0.15\,\text{m/s}$, and $\dfrac{dx}{dt} = 0.2\,\text{m/s}$ when $x = 3\,\text{m}$. Differentiating implicitly with respect to t, we get $x^2 + y^2 = L^2 \quad \Rightarrow \quad 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0 \quad \Rightarrow \quad y\dfrac{dy}{dt} = -x\dfrac{dx}{dt}$. Substituting the given information gives us $y(-0.15) = -3(0.2) \quad \Rightarrow \quad y = 4\,\text{m}$. Thus, $3^2 + 4^2 = L^2 \quad \Rightarrow$ $L^2 = 25 \quad \Rightarrow \quad L = 5\,\text{m}$.

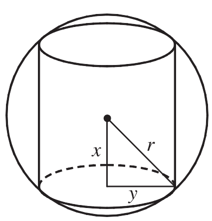


10 Let b be the length of the base of the box and h the height. The volume is $32{,}000 = b^2h \quad \Rightarrow \quad h = 32{,}000/b^2$.

The surface area of the open box is $S = b^2 + 4hb = b^2 + 4(32{,}000/b^2)b = b^2 + 4(32{,}000)/b$.

So $S'(b) = 2b - 4(32{,}000)/b^2 = 2\left(b^3 - 64{,}000\right)/b^2 = 0 \quad \Leftrightarrow \quad b = \sqrt[3]{64{,}000} = 40$. This gives an absolute minimum since $S'(b) < 0$ if $0 < b < 40$ and $S'(b) > 0$ if $b > 40$. The box should be $40 \times 40 \times 20$.

11



The cylinder has volume $V = \pi y^2(2x)$. Also $x^2 + y^2 = r^2 \quad \Rightarrow \quad y^2 = r^2 - x^2$, so

$V(x) = \pi(r^2 - x^2)(2x) = 2\pi(r^2x - x^3)$, where $0 \le x \le r$.

$V'(x) = 2\pi\left(r^2 - 3x^2\right) = 0 \quad \Rightarrow \quad x = r/\sqrt{3}$. Now $V(0) = V(r) = 0$, so there is a maximum when $x = r/\sqrt{3}$ and $V\left(r/\sqrt{3}\right) = \pi(r^2 - r^2/3)\left(2r/\sqrt{3}\right) = 4\pi r^3/\left(3\sqrt{3}\right)$.