Math 231 Review

1. Determine whether the statement is true or false.

(1) If $\lim_{x\to 5} f(x) = 2$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

(2) If $\lim_{x\to 5} f(x) = 0$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

(3) If f is a polynomial, then $\lim_{x\to a} f(x) = f(a)$.

(4) A function can have two different horizontal asymptotes.

(5) If the line $x = 1$ is a vertical asymptote of $y = f(x)$, then f is not defined at 1.

(6) If f is continuous at a, then f is differentiable at a.

(7) If f and g are differentiable, then $[f(x) + g(x)]' = f'(x) + g'(x)$.

(8) If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.

(9) If f has an absolute minimum value at c, then $f'(c) = 0$.

(10) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$.

(11) If f and g are increasing on an interval I, then $f + g$ is increasing on I.

(12) $\int_a^b [f(x)g(x)]dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$.

(13) $\int_a^b xf(x)dx = x\int_a^b f(x)dx$.

(14) All continuous functions have antiderivatives.

(15) $\tan^{-1}(-1) = \frac{3\pi}{4}$.

(16) $\pi^{\sqrt{5}} = e^{\sqrt{5}\ln\pi}$.

(17) $\ln(x + y) = \ln x + \ln y$.

(18) $(10^x)' = x10^{x-1}$.

2. Find the limit.

(1) $\displaystyle\lim_{x\to -3} \frac{x^2 - 9}{x^2 + 2x - 3}$

(2) $\displaystyle\lim_{x\to -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^4}$

(3) $\displaystyle\lim_{x\to \infty} (\sqrt{x^2 + 4x + 1} - x)$

(4) $\displaystyle\lim_{s\to 16} \frac{4 - \sqrt{s}}{s - 16}$

(5) $\displaystyle\lim_{x\to 0} \cos(x + \sin x)$

3. Differentiate the function.

(1) $y = \dfrac{x^2 - x + 2}{\sqrt{x}}$

(2) $y = x^2 \sin \pi x$

(3) $y = \dfrac{\tan x}{1 + \cos x}$

(4) $y = \sin(\cos x)$

(5) $y = \ln(x \ln x)$

(6) $y = e^{\tan(2\theta)}$

(7) $y = \dfrac{e^t}{1 + e^t}$

(8) $y = (\cos x)^x$

(9) $y = 3^{x \ln x}$

(10) $y = \sqrt{\arctan x}$

(11) $y = (\arcsin(2x))^2$

(12) $y = \sin^{-1}(e^x)$

4. Find an equation of the tangent line to the curve at the given point.

(1) $y = \sqrt{1 + 4\sin x},\ (x, y) = (0, 1)$

(2) $x^2 + 4xy + y^2 = 13,\ (x, y) = (2, 1)$

5. Let $f(x) = 200 + 8x^3 + x^4$.

(1) Find the intervals of increase or decrease.

(2) Find the local maximum and minimum values.

(3) Find the intervals of concavity and the inflection points.

(4) Use (1)-(3) to sketch the graph of f.

6. Evaluate the integral.

(1) $\int \frac{1+t+t^2}{\sqrt{t}} dt$

(2) $\int \frac{x^5 - x^4 + 2x}{x^4} dx$

(3) $\int_0^1 y(y^2+1)^5 dy$

(4) $\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t dt$

(5) $\int \sin(\pi t)\cos(\pi t) dt$

(6) $\int \frac{x+1}{x^2+2x} dx$

(7) $\int_0^4 \frac{1}{16+t^2} dt$

(8) $\int \frac{x}{\sqrt{1-x^2}} dt$

(9) $\int_0^1 \frac{1}{e^{\pi x}} dx$

(10) $\int_0^3 \frac{1}{5x+1} dx$

(11) $\int \frac{\cos x}{2+\sin x} dx$

7. A particle is moving with the given data. Find the position fo the particle.

(1) $a(t) = 10\sin t + 3\cos t,\ s(0) = 0,\ s(2\pi) = 12.$

(2) $a(t) = t^2 - 4t + 6,\ s(0) = 0,\ s(1) = 20.$

8. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

9. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

10. A box with a square base and open top must have a volume of $32,000$ cm^3. Find the dimensions of the box that minimize the amount of material used.

11. A right circular cylinder is inscribed in a sphere of radius 1. Find the largest possible volume of such a cylinder.