1. Recall that

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

Since $f'(x) = 3x^2 + 1$, we have $f'(-1) = 4$ and $f(-1) = 1$. Therefore

$$x_2 = -1 - \frac{1}{4} = -1.25$$

2. By antidifferentiation, we see that f must be of the form

$$f(x) = \frac{x^3}{3} + \cos x + C.$$

Using the condition $f(0) = 0$, we get $C = -1$. So

$$f(x) = \frac{x^3}{3} + \cos x - 1.$$

3. Denote the dimension of the square base by x and the height of the box by y. We need to minimize the area of the box (with open top)

$$A = x^2 + 4xy$$

subject to the volume constraint

$$V = x^2 y = 4000.$$

Solving from the constraint we get

$$y = \frac{4000}{x^2}.$$

Therefore we can write

$$A(x) = x^2 + 4x\frac{4000}{x^2} = x^2 + \frac{16000}{x}$$

where $x > 0$ can be any positive real number.

To minimize $A(x)$ we find the critical number(s) by solving

$$A'(x) = 2x - \frac{16000}{x^2} = 0.$$

This equation gives a unique critical number $x = \sqrt[3]{8000} = 20$. One can easily verify, for example by the 1st derivative test, that $A(x)$ attains an absolute minimum at $x = 20$.

To conclude, when the dimensions $x = 20$ cm and $y = \frac{4000}{20^2} = 10$ cm, the material for making the box is minimized.