

1. (a) The area represented by  $\int_{-1}^1 2|x|dx$  consists of two right triangles above the  $x$ -axis with height 2 and base 1. Therefore the integral is equal to  $\boxed{2}$ .

(b) Note that the graph of  $y = \sqrt{4 - x^2}$  is the first quadrant of the circle centred at  $(0, 0)$  and of radius 2. Therefore the graph of  $\sqrt{4 - x^2} + 2$  is such a graph shifted up by 2, and the area represented by  $\int_0^2 (\sqrt{4 - x^2} + 2)dx$  is equal to the area of the sector plus the area of a  $2 \times 2$  square, that is  $\boxed{\pi + 4}$ .

2.

$$\begin{aligned}
 (a) \quad & \int_0^1 6x(1 + x^2)dx \\
 &= \int_0^1 6x + 6x^3 dx \\
 &= \left[ 6\frac{x^2}{2} + 6\frac{x^4}{4} \right]_0^1 \\
 &= \left[ 3x^2 + 3\frac{x^4}{2} \right]_0^1 \\
 &= 3 + \frac{3}{2} \\
 &= \boxed{\frac{9}{2}}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad & \int_1^4 \frac{\sqrt{x} - 4}{x^2} dx \\
 &= \int_1^4 \frac{\sqrt{x}}{x^2} - \frac{4}{x^2} dx \\
 &= \int_1^4 x^{-3/2} - 4x^{-2} dx \\
 &= \left[ -2x^{-1/2} + 4x^{-1} \right]_1^4 \\
 &= \left[ -\frac{2}{\sqrt{x}} + \frac{4}{x} \right]_1^4 \\
 &= (-1 + 1) - (-2 + 4) \\
 &= \boxed{-2}
 \end{aligned}$$

3. (a) The displacement equals the integral of  $v(t)$  over  $[0, \pi]$ , i.e.

$$\int_0^\pi (2 \cos t - 1)dt = [2 \sin t - t]_0^\pi = \boxed{-\pi}$$

(b) The distance traveled equals the integral of  $|v(t)|$  over  $[0, \pi]$ . Notice that  $v(t) > 0$  if  $t < \pi/3$ , and  $v(t) < 0$  if  $t > \pi/3$ . Therefore

$$\int_0^\pi |v(t)|dt = \int_0^{\pi/3} v(t)dt + \int_{\pi/3}^\pi -v(t)dt$$

where

$$\int_0^{\pi/3} v(t) dt = [2 \sin t - t]_0^{\pi/3} = \sqrt{3} - \frac{\pi}{3}$$

and

$$\int_{\pi/3}^{\pi} v(t) dt = [2 \sin t - t]_{\pi/3}^{\pi} = (0 - \pi) - (\sqrt{3} - \frac{\pi}{3}).$$

So

$$\int_0^{\pi} |v(t)| dt = 2(\sqrt{3} - \frac{\pi}{3}) + \pi = \boxed{2\sqrt{3} + \frac{\pi}{3}}$$