1.

$$\ln(2\sqrt{e}) - \ln 2 = \ln 2 + \ln \sqrt{e} - \ln 2 = \ln \sqrt{e} = \frac{1}{2} \ln e = \boxed{\frac{1}{2}}$$

2. Taking the natural logarithms of both sides, we get

$$\ln(e^{-x^2}) = \ln \frac{1}{2}$$

that is

$$-x^2 = -\ln 2$$

or

$$x = \boxed{\pm\sqrt{\ln 2}}$$

3. (a)

$$\boxed{\cot x}$$

(b)

$$\boxed{\frac{2e^{-1/x^2}}{x^3}}$$

4.

$$\boxed{\ln(1 + e^x) + C}$$

Bonus.

$$\boxed{\frac{x(x-1)^2}{\sqrt{x^2+1}} \left(\frac{1}{x} + \frac{2}{x-1} - \frac{x}{x^2+1} \right)}$$