1(a). $f(x) = \sqrt{x} - x^{3/2}$. So the antiderivatives of f are

$$F(x) = \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C.$$

1(b). $f(x) = x - 2 + x^{-1/2}$. So the antiderivatives of f are

$$F(x) = \frac{1}{2}x^2 - 2x + 2x^{1/2} + C.$$

2. Denote the dimension of the square base by x and the height of the box by y. We need to minimize the area of the box (containing the top)

$$A = 2x^2 + 4xy$$

subject to the volume constraint

$$V = x^2 y = 16000.$$

Solving from the constraint we get

$$y = \frac{16000}{x^2}.$$

Therefore we can write

$$A(x) = 2x^2 + 4x\,\frac{16000}{x^2} = 2x^2 + \frac{64000}{x}$$

where $x > 0$ can be any positive real number.

To minimize $A(x)$ we find the critical number(s) by solving

$$A'(x) = 4x - \frac{64000}{x^2} = 0.$$

This equation gives a unique critical number $x = \sqrt[3]{16000} = 20\sqrt[3]{2}$. One can easily verify, for example by the first derivative test, that $A(x)$ attains an absolute minimum at $x = 20\sqrt[3]{2} \approx 25.198$.

To conclude, when the dimensions $x = 20\sqrt[3]{2}$ cm and $y = \frac{16000}{x^2} = 20\sqrt[3]{2}$ cm, the material for making the box is minimized.