Solution:

1. Denote the dimension of the square base by x and the height of the box by y. We need to minimize the area of the box (with open top)

$$A = x^2 + 4xy$$

subject to the volume constraint

$$V = x^2 y = 4000.$$

Solving from the constraint we get

$$y = \frac{4000}{x^2}.$$

Therefore we can write

$$A(x) = x^2 + 4x\frac{4000}{x^2} = x^2 + \frac{16000}{x}$$

where $x > 0$ can be any positive real number.

To minimize $A(x)$ we find the critical number(s) by solving

$$A'(x) = 2x - \frac{16000}{x^2} = 0.$$

This equation gives a unique critical number $x = \sqrt[3]{8000} = 20$. One can easily verify, for example by the 1st derivative test, that $A(x)$ attains an absolute minimum at $x = 20$.

To conclude, when the dimensions $x = 20$ cm and $y = \frac{4000}{20^2} = 10$ cm, the material for making the box is minimized.