6.2#2 For $\varepsilon \in (0,1)$, let P be the partition

$$P = \left\{0, 1 \pm \frac{\varepsilon}{8}, 2 \pm \frac{\varepsilon}{8}, 3 \pm \frac{\varepsilon}{8}, 4 - \frac{\varepsilon}{8}\right\}.$$

By direct computation, we have

$$U(P,f) - L(P,f) = \frac{3\varepsilon}{4} + \frac{\varepsilon}{8} < \varepsilon.$$

6.2#4 Since

$$L(P_n, f) \le \underline{\int_a^b} f \le \overline{\int_a^b} f \le U(P_n, f)$$

and

$$\lim_{n\to\infty} \big(U(P_n,f) - L(P_n,f)\big) = 0,$$

by the Squeeze Theorem (v2), we have

$$\underline{\int_a^b} f = \overline{\int_a^b} f = \lim_{n\to\infty} L(P_n,f) = \lim_{n\to\infty} U(P_n,f).$$

This proves $f \in R[a,b]$ and the claimed limits.