6.1#6 Let $P = \{x_k\}_{k=1}^n$ be any partition of $[a,b]$. For $k = 1, \cdots, n$, we have, since $f(x) \geq 0$ for all $x \in [a,b]$,

$$m_k = \inf_{[x_{k-1},x_k]} f \geq 0.$$

It then follows that

$$L(P,f) = \sum_{k=1}^n m_k \Delta x_k \geq 0.$$

Since P is arbitrary, we get

$$\underline{\int_a^b} f = \inf L(P,f) \geq 0.$$

This completes the proof.

6.3#5(a) The assumption $f \in R[a,b]$ implies that f is bounded. Therefore there exists $M > 0$ such that $f([a,b]) \subset [-M,M]$. Let $g(x) = |x|$. Then g is continuous on $[-M,M]$. So, by Theorem 6.3.4, $|f| = g \circ f \in R[a,b]$.
To prove $\left|\int_a^b f\right| \leq \int_a^b |f|$, notice that either $\int_a^b f \geq 0$ or $\int_a^b f < 0$. In the first case, we have

$$\left|\int_a^b f\right| = \int_a^b f \leq \int_a^b |f|$$

where the last inequality follows from $f(x) \leq |f(x)|,\ \forall x \in [a,b]$ and Theorem 6.3.2. In the second case, we have

$$\left|\int_a^b f\right| = -\int_a^b f = \int_a^b (-f) \leq \int_a^b |f|$$

where we have used Theorem 6.3.1(b) and $-f(x) \leq |f(x)|,\ \forall x \in [a,b]$. So, in both cases we have

$$\left|\int_a^b f\right| \leq \int_a^b |f|,$$

and the proof is complete.

6.3#9 Let $h = g - f$. Then by the assumption we have $h(x) = 0$ except for finitely many $x \in [a,b]$. By Exercise 6.2.13, this implies $h \in R[a,b]$ and

$$\int_a^b h = 0.$$

By Theorem 6.3.1(a), it follows that

$$g = f + h \in R[a,b]$$

and

$$\int_a^b g = \int_a^b (f+h) = \int_a^b f + \int_a^b h = \int_a^b f.$$

This proves the claims.