3.3#13(a) Since f is an odd function, by definition we have

$$f(-x) = -f(x).$$

Taking the limit of both sides as $x \to 0$, we get

$$\lim_{x\to 0} f(-x) = -\lim_{x\to 0} f(x) = -L.$$

On the other hand, $x \to 0$ iff $-x \to 0$, so

$$\lim_{x\to 0} f(-x) = \lim_{x\to 0} f(x) = L.$$

Thus

$$L = -L$$

which implies $L = 0$.

4.1#5 Since f is continuous at $x = a$, by definition we have

$$\lim_{x\to a} f(x) = f(a).$$

However, this implies

$$\lim_{x\to a} |f(x)| = |f(a)|,$$

which by definition shows that $|f|$ is continuous at $x = a$.

4.3#1(a)

(i) $f(x) = 1/x$ is continuous on the bounded, open interval $(0, 1)$. But f is unbounded.

(ii) $f(x) = x$ is continuous on the unbounded, closed interval $[0, \infty)$. But f is unbounded.

(iii) Define $f(x) = 1/x$ if $0 < x \leq 1$ and $f(x) = 0$ if $x = 0$. Then $f(x)$ is a discontinuous function on the closed interval $[0, 1]$, meanwhile f is unbounded.

4.3#11 Since f is continuous and nonvanishing on $[a, b]$, by **Exercise 4.1#5** above, so is $|f|$. On the other hand, by Theorem 4.1.8(c), the function $1/|f|$ is also continuous on $[a, b]$. Thus, by Theorem 4.3.4, $1/|f|$ must be bounded on $[a, b]$, i.e. we have

$$\frac{1}{|f(x)|} \leq M, \ \forall x \in [a, b]$$

for some positive number M. Consequently,

$$|f(x)| \geq \frac{1}{M}, \ \forall x \in [a, b].$$

This shows that f is bounded away from zero on $[a, b]$.