Math 231 Worksheet 5

1. Find the absolute maximum and absolute minimum of f on the given interval.

$$f(x) = 2x^3 - 3x^2 + 4, \quad [-1, 2].$$

2. A box with a square base and open top must have a volume of 4000 cm^3. Find the dimensions of the box that minimize the amount of material used.

Solutions:

1. To find the critical numbers, set

$$f'(x) = 6x^2 - 6x = 0$$

that is,

$$6x(x-1) = 0.$$

Therefore $x = 0$ and $x = 1$ are the critical numbers. The corresponding critical points are $(0, 4)$ and $(1, 3)$. Next, the endpoints are $(-1, -1)$ and $(2, 8)$.

So the maximum of f is $\max\{4, 3, -1, 8\} = 8$, attained at $x = 2$; the minimum of f is $\max\{4, 3, -1, 8\} = -1$, attained at $x = -1$.

2. Denote the dimension of the square base by x and the height of the box by y. We need to minimize the area of the box (with open top)

$$A = x^2 + 4xy$$

subject to the volume constraint

$$V = x^2 y = 4000.$$

Solving from the constraint we get

$$y = \frac{4000}{x^2}.$$

Therefore we can write

$$A(x) = x^2 + 4x\frac{4000}{x^2} = x^2 + \frac{16000}{x}$$

where $x > 0$ can be any positive real number.

To minimize $A(x)$ we find the critical number(s) by solving

$$A'(x) = 2x - \frac{16000}{x^2} = 0.$$

This equation gives a unique critical number $x = \sqrt[3]{8000} = 20$. One can easily verify, for example by the 1st derivative test, that $A(x)$ attains an absolute minimum at $x = 20$.

To conclude, when the dimensions $x = 20$ cm and $y = \frac{4000}{20^2} = 10$ cm, the material for making the box is minimized.